International Study of Achievement in Mathematics (Belgium, England, Finland, France, Netherlands, Israel, Japan, Scotland, Sweden, United States, Germany, Australia)

1964
INTERNATIONAL EDUCATIONAL ACHIEVEMENT STUDY

MANUAL FOR LOCAL COORDINATORS
I. **Introduction**

Educational Research centers from eleven different countries have on their own initiative joined together in this International Project for the Evaluation of Educational Achievement (I.E.A.). These countries are: Belgium, England, Federal Republic of Germany, Finland, France, Israel, Japan, The Netherlands, Scotland, Sweden and U.S.A. The project is being carried out under the auspices of the UNESCO Institute for Education, Hamburg.

This is one of the first major cross-national studies in comparative education which uses an empirical approach. The aim of the project is to study in a systematic way the educational outcomes of the different school systems based on such factors as organization, curriculum and teaching method. Another aspect of the study is to analyze the findings in view of a specific country's background and aims.

The first subject area undertaken is that of mathematics. The mathematical performance of various groups of students at different points in their respective school systems will be measured. The study may clarify the role of mathematics in the schools of modern technological societies and may bring out the importance of various factors in mathematical learning. The results may have important implications for the teaching of mathematics in all countries.

The plans for the research require representative samples of pupils aged 13 to 14 years to be tested in each country and further representative samples to be taken at the different terminal points of secondary education.

In addition to testing the mathematical performance, it is also important to assess the attitudes toward the learning of mathematics. Therefore we are administering the following supplementary questionnaires:

1) School questionnaire which is an inquiry about the organization of the school.

2) Teacher questionnaire which deals with the professional background of the mathematics teachers and the mathematics curriculum in their school.
3) Student questionnaire which asks for
   a) demographic data pertaining to the student and his parents, and
   b) the student’s academic performance and his future educational and
      vocational plans.

4) Student opinion booklet which deals with student opinions concerning their mathe-
   matics instruction and students’ attitudes toward mathematics and related areas.

   An investigation like the present one gives countries an opportunity to learn from
   each other on the basis of systematized facts. The study does not aim at making crude
   comparisons in terms of average mathematical competence at certain age levels, nor does
   it aim at criticizing existing practices.

II. **Target Population**

   The mechanics of organizing an investigation of this kind are necessarily complex. 
Essentially, a number of "target populations" of students at different levels of secondary 
schooling has been defined and, in each country, samples of students in each of these 
populations have been selected. The sampling scheme has been designed so that every 
school containing students in each of the defined populations had the same chance of being 
in the final sample. Random choice alone actually determined which schools and hence 
which students are to be tested.

   For administrative purposes in this country, brief operational definitions of the 
groups to be tested are given in Table 1, along with the tests and other materials to be 
given to each group.

III. **Testing Materials**

   The following materials will be required for each group of students:

   a) Students Name Forms
### TABLE 1

<table>
<thead>
<tr>
<th>Group Number</th>
<th>Population Definition</th>
<th>Tests to be Given</th>
<th>Student Opinion Booklet</th>
<th>Student Questionnaire</th>
<th>Answer Sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Students who are 13+ years* and not in the 8th grade</td>
<td>A, B, C</td>
<td>OP</td>
<td>ST 1</td>
<td>13+</td>
</tr>
<tr>
<td>2</td>
<td>Students who are 13+ years* and in 8th grade</td>
<td>A, B, C</td>
<td>OP</td>
<td>ST 1</td>
<td>13+</td>
</tr>
<tr>
<td>3</td>
<td>Students who are not 13+ years* but in 8th grade</td>
<td>A, B, C</td>
<td>OP</td>
<td>ST 1</td>
<td>13+</td>
</tr>
<tr>
<td>4</td>
<td>Students who are in 10th grade</td>
<td>3, 4, 5</td>
<td>OP</td>
<td>ST 2</td>
<td>15+</td>
</tr>
<tr>
<td>7</td>
<td>Students who are in 12th grade and currently taking Mathematics</td>
<td>5, 7, 8, 9</td>
<td>OP</td>
<td>ST 2</td>
<td>17M</td>
</tr>
<tr>
<td>8</td>
<td>Students who are in 12th grade and not currently taking Mathematics</td>
<td>3, 5, 6</td>
<td>OP</td>
<td>ST 2</td>
<td>17NM</td>
</tr>
</tbody>
</table>

* Born between April 2, 1950 - April 1, 1951.

b) A set of "Practice Questions" sheets.
c) A set of mathematics tests.
d) A set of Student Opinion booklets.
e) A set of answer sheets.
f) A set of Student Questionnaire forms.
g) A set of Return Envelopes
h) Copies of Manual for Test Administrators
i) A set of test record forms.

As soon as the appropriate sets of materials for your school have been received from us, please check them carefully against the enclosed invoice to see that everything is there. If there are any discrepancies, please contact us immediately.

You are responsible for the return of all materials sent to you. Please keep the materials locked up when not in use and be sure to keep track of all test blanks, questionnaires, and other materials. Make sure that all of them are returned to you after each
period of testing. All tests, answer sheets, Student Opinion Booklets, and Student Questionnaires, used and unused, should be returned to us promptly when testing has been completed. (See section VII B.)

IV. **Testing Schedule**

It is important that the timetable for the administration of the tests and questionnaires be followed as closely as possible. In any event the order of administration—tests first, Student Opinion Booklets second and Student Questionnaires third—should be strictly adhered to. For ease of reference in the remainder of this document, Day 1 refers to any day or days on which mathematics testing is carried out. Day 2 refers to the day or days on which the Student Opinion Booklets and Student Questionnaires are given.

A suggested schedule for administration is shown below.

| TABLE II |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | Level 1          | Level 2          | Level 3A         | Level 3B         | Time            |
| DAY I           | Groups 1, 2, 3   | Group 4          | Group 7          | Group 8          |                 |
|                 |                 |                 |                 |                 |                 |
| Morning         | Test A           | Test 3           | Test 5           | Test 3           | 60 min.         |
|                 | Break            | Break            | Break            | Break            |                 |
|                 | Test B           | Test 4           | Test 7           | Test 5           | 60 min.         |
| Afternoon       | Test C           | Test 5           | Test 8           | Test 6           | 60 min.         |
|                 | Break            |                  |                  |                  |                 |
|                 |                  |                  |                  |                  |                  |
|                 |                  |                  |                  |                  |                  |
|                 |                  |                  |                  |                  |                  |
| DAY II          |                 |                  |                  |                  |                 |
| Morning         | Student Opinion Booklet |           | Student Questionnaire | 20-30 min.     |                 |
|                 |                  |                  |                  |                  |                 |
V. Students Name Forms

A separate form will be prepared for the students in each group to be tested. This form will contain the names of those selected to be tested. No other students should be tested. If a student has left the school since the names were selected, please draw a red line through the name. Do not add any further names. This form is to be retained during the testing and further information added before it is returned to us. (See section VII A.)

VI. General Instructions for Administration

A. In order to keep conditions as standard as possible, it is important NOT to tell the students in advance that they will be taking these tests. If they do know in advance that they are to be tested, some absenteeism or pre-test anxiety is bound to occur; the effect this will have on the results is unpredictable.

B. To obtain reliable results, conditions throughout all schools tested should be as uniform as possible. Students should be put at their ease. It is important that both the general and detailed instructions contained in this manual and in the Manual for Administrators be scrupulously followed.

C. The ideal room for testing is one in which students will be comfortable and free from all distractions. The students should be seated sufficiently well-separated from one another so that there will be little opportunity or inclination to seek help from others.

D. If possible, there should be two proctors to each room. One of them should be directly responsible for administering the tests. The other should monitor the room quietly, and insure that answers are given in the correct places and in the correct way. However, no help should be given to students on any of the problems.

If more than 25 or 30 students are being tested at the same time, it will be desirable to have one additional assistant to help with passing out papers and proctoring the tests.

E. The tests are to be answered in pencil. It is therefore necessary for a sufficient supply of sharpened pencils to be available. Scratch paper and erasers are also required.
F. The Manual for Test Administrators gives the full details necessary for the actual administration of the tests. It is important that this manual also should be studied by all test administrators.

VII. Post-Testing Procedures

A. Problem of Absentees: It is hoped that all students who were present on Day 1 and who then completed the mathematics tests, will also complete the Student Opinion Booklet and the Student Questionnaire. It is requested, therefore, that if at all possible, any student present on Day 1 but absent on Day 2, should be subsequently given the Day 2 material.*

Students in the sample who were present on Day 2, but who were absent for Day 1, should be given the Day 2 material, but they should not be asked to take the mathematics tests.

The remaining students in the sample, who were absent on both days, should be given, if at all possible, the Day 2 material.

The students can be divided into the following six possible categories:

1. Students present on both Day 1 and Day 2 and who have completed all material.

2. Students present on Day 1, absent on Day 2, but who have subsequently completed Day 2 material.

3. Students present on Day 1, absent on Day 2, and who have not subsequently been given Day 2 material.

4. Students absent on Day 1, present on Day 2, and who have completed Day 2 material only.

5. Students absent on both Day 1 and Day 2, but who have subsequently completed Day 2 material.

6. Students absent on both Day 1 and Day 2, and who have not subsequently completed Day 2 material.

*If the math tests are given on more than one day, it is hoped that make-ups will be given for students who miss a part of the test.
The appropriate category number into which a student falls should be entered in the column on the Students Name Forms headed "Absentee Category."

B. Procedures for Returning Materials: The return envelopes for the students in each group should now be arranged in the order of the names on the Names Form for that group. Each group of envelopes together with its appropriate Names Form should then be bound together for return. It is important that the envelopes should not be folded or bent in any way. Care should be taken that they are packed so that the edges will not be torn or frayed.

The package of return envelopes, together with all the mathematics tests and Student Opinion Booklets should now be securely packed and returned to us at the following address:

International Educational Achievement Study
Institute of Psychological Research
Teachers College, Columbia University
New York, N.Y. 10027

PLEASE CHECK THAT ALL MATERIALS RECEIVED HAVE BEEN RETURNED.
INTERNATIONAL EDUCATIONAL ACHIEVEMENT STUDY

MANUAL FOR TEST ADMINISTRATORS
It is assumed that all Test Administrators will have acquainted themselves with the Manual for Local Coordinators.

A. General Instructions

1. The tests and questionnaires should be answered in pencil. Supply students with a sharpened pencil and eraser. Extra pencils should be available.

2. Provide enough scratch paper for the mathematics tests. The test booklet and the answer sheet are not to be used for figuring.

3. No rulers, compasses, protractors, tables of functions, or other equipment should be allowed.

4. Time mathematics tests carefully. The time limit is 60 minutes. Do not give any extra time.

5. There is no time limit on the Student Opinion Booklet or the Student Questionnaire. They will take approximately 20-30 minutes each to complete.

   No help should be given in answering the Student Opinion Booklet, but some help may be given in answering the Questionnaire.

B. Detailed Instructions

   DAY 1

1. Put the following sample of types of answers on the blackboard:

   Types of Answers

   1. A ==== B ==== C ==== D ==== E ====

   2.  - - - - - - - - - - - - - - - - - - -
   A ==== B ==== C ==== D ==== E ====

2. Have the appropriate Mathematics tests ready. Do not tell students that later they are to be given the Student Opinion Booklet and Questionnaire.

3. After the students are seated, read the following instructions verbatim. Make no attempt to memorize them.

   "This school has been chosen as one of those to represent the United States in an international project to study what young people know about mathematics. Different countries from all over the world are taking part in this study."
"You will probably find some parts of the test easy; some you are likely to find hard. Do your best on all problems. Listen carefully to the instructions as they are given. Follow them exactly. Do the best you can."

4. Now pass out to each student a copy of the first test to be given and an answer sheet, with the name side uppermost. While you are reading the following instructions, have the proctor check to see that the blanks are being filled in correctly.

Now say:

"On line 1 write your name, putting your last name first. On line 2 write the name of your school. On line 3 write your grade. On line 4 write the date of your birth (month, day and year). Do not open the test booklet until you are told to do so."

5. Next say:

"For this test we shall use a special answer sheet that is separate from the test itself. With this answer sheet, all the papers can be scored very quickly and accurately by a special machine. It is important that you listen carefully to the instructions about marking the answer sheet."

"The questions on the test are to be answered in two different ways. Sometimes several answers are given for a question and you are to choose the best one. On other questions you have to find the answer and write it on a line on the answer sheet. Look at the sample that I have put on the board. For the first question the answer is to be selected from the five choices, named A, B, C, D, and E. If you think the correct answer to the first question is "C", you should make a solid pencil mark between the dotted lines at the right of "C", like this. (Mark between the dotted lines at the right of "C" on the board.) If you change your mind and want to change an answer, erase the first mark completely and then mark your other choice. The second example shows a line on which you are to write your answer. If question 2 had asked: 'How much is 2 times 3?' you
would write the number 6 on the answer line, like this." (Demonstrate by writing 6 on the answer line.)

Then say:

"In questions like this you do not make any marks between the dotted lines directly underneath your answer. They will be used by the person who scores your test. Are there any questions about the way you are to mark the answers?" (Answer any questions. Try to be sure that all students understand how they are to mark their answers.)

6. Continue as follows:

"Now take the sheet headed 'Practice Questions'. Read the directions and mark the answers on the bottom of the answer sheet, on the same side with your name in the section marked PT. If you have any questions about marking the answers to these problems, raise your hand for help." (Answer any questions, and help students to mark the answers correctly.)

"Turn now to your test booklet and read the directions to yourself while I read them out loud.

'This test samples many different topics in mathematics. Some problems you will probably know how to do, and some you will probably not know how to do. Do the very best you can, but do not waste time puzzling over any one question. If you get stuck on a question skip it and go on to the next one. If you finish the test before time is up, you can go back and work on any questions that you have skipped.

'Do not make any marks on the test booklet. Use the separate scratch paper for your figuring. Be sure to mark the correct row on your answer sheet for each question.'"

7. Continue as follows:

"Now turn over your answer sheet. The answers on this test are to be marked in the section which says '_________'. (Hold up the answer sheet and indicate the appropriate test T-A, T-B, T-3, etc.)
Then say:

"If your pencil breaks, please put up your hand immediately and I will give you another one. You will have 60 minutes for the test. Remember, do not waste time if you cannot do a problem; skip it and go on to the next one. Now open your test booklet and start working." (Note the time when the group starts.)

After 30 minutes say:

"About half the time is over. Remember, do not waste time on the problems you don't know how to do."

After 50 minutes, say:

"You have about 10 minutes left. Look at any problems you have left out and see if you can do some of them."

After 60 minutes, say:

"Time is up. Stop working and put your pencils down. Your papers will now be collected."

8. First collect the test booklets and count them to be sure that you have all of them. Enter the number on the Test Record Form. If the number checks with the number received, no further check is necessary. If the count is short, check the serial numbers to find which is/are missing, and take steps to collect those booklets. If another test is to be given directly after this one, the answer sheets may be left with the students. If not, collect the answer sheets. It is essential that when storing the answer sheets, they are not bent or folded in any way. Now collect the scratch paper.

9. If another test is to be given in the same session, it is suggested that the examinees be given a chance to stand, stretch, and relax for a minute or two. If the local schedule permits, it may be desirable to give students a 5 or 10 minute intermission, and to permit them to leave the room, making sure that no test materials are removed. Have them leave the answer sheets turned over on their desks. After the intermission, hand out the new test booklets.
Then say:

"Here is another test. Answer the problems in the same way as you did on the test you took before. Turn your answer sheets over." (When all pupils are ready to start, continue with the instructions given under Section 7.)

10. If tests are to be given on different days, in passing out answer sheets, be sure that each student has his own answer sheet. It is suggested that the answer sheets be passed out with the name side uppermost. Continue with the detailed instructions in Part B, omitting Section 4. Also, it will not be necessary to do the practice examples. Proceed through Section 8.

11. When all the mathematics tests for the group have been given, the test booklets should be stored in a safe place until the rest of the testing has been completed. The answer sheets, which are also to be used for the Student Opinion Booklet, should also be kept in a safe place until Day 2. Scratch paper should be destroyed.

**DAY 2**

1. On this day, have the following materials ready:

   a) Student Opinion Booklets
   b) Student Questionnaires
   c) Students' answer sheets
   d) Return envelopes

2. When the students are seated, give each student his own answer sheet, a copy of the Student Opinion Booklet and a return envelope.

Now say:

"In the many countries engaged in this study, the students are also being asked to give their opinions and views about mathematics, school, and other topics. The purpose of the booklet we just passed out to you is to find out what students think about a number of things. You should give answers to these statements as rapidly and as accurately as you can. If you are not taking any mathematics this semester, answer the statements about mathematics on the basis of your most recent mathematics course."
There are no 'right' or 'wrong' answers to these statements, so please answer according to your own opinions. Your answers will be used only for the study and will have no effect on your school marks. Later you will seal your answer sheets in the envelopes we just passed out to you. Your answers to these statements should be marked in the section of your answer sheet labelled SO." (Holding up the answer sheet, indicate the section where the answers are to be marked.)

"The numbers in this section of the answer sheet correspond to the numbered statements in this Student Opinion Booklet. In answering statements 1 through 22, read each statement carefully and mark:

Answer space A--if the statement is usually TRUE about your school.

Answer space B--if the statement is usually NOT TRUE about your school.

Answer space C--if you cannot decide whether the statement is usually TRUE or NOT TRUE about your school.

Slightly different directions are given later for answering statements 23 through 65. When you come to these, read the directions carefully and then continue. Are there any questions?" (Answer questions concerning the method of recording the responses.)

Then say:

"You will be given sufficient time to read and answer all the statements, but do not waste time. Open your booklets and begin." (Have proctors walk around the room, checking to make sure that every student fills in his answers to the Student Opinion Booklet in the correct place.)

When most students have completed the booklet, say:

"After you have finished, check to see that all your responses are in the correct place on your answer sheet." (Wait until everybody finishes.)

Now say:

"Take your envelope, and on it print your name clearly in capital letters. Now put your answer sheet in the envelope. Do not seal your
envelopes yet." (Collect the Student Opinion Booklets, but not the return envelopes.)

3. Next, hand out the Student Questionnaires, and say:

"In order that the results of this study can be accurately assessed, it is necessary to know certain things about each student who has been tested. You are asked, therefore, to fill out this Student Questionnaire as carefully as you can. Again, I should remind you that your responses will be regarded as confidential. If you cannot understand a question, please raise your hand. Now start answering the questionnaire."

Help may be given to any student who needs it. It will be appreciated if the information requested in this Questionnaire is as accurate as possible. Every assistance should be given to the students to attain this end.

When all have completed the Student Questionnaire, say:

"Put your Student Questionnaire in your envelope and seal the envelope."

The envelopes should be collected, and returned to the testing coordinator for your school or school system, together with the testing materials. The testing coordinator will forward them to the Institute of Psychological Research, Teachers College, Columbia University, New York 27, N. Y.
TODAY'S DATE

Month      Day      Year

1. Name of your school

2. Name (PRINT)  Last    First   Middle

3. Grade in school

4. Sex (CHECK ONE) Boy    Girl

5. Date of birth:  Month   Day   Year

6. CHECK ONE: Your present school, course or program is primarily

   a) academic (preparing you for college, university or
      other institution of higher learning)

   b) vocational (preparing you to enter industry or
      commerce, i.e. to get a specific job)

   c) general (a general course directed toward
      neither a specific job nor further education)

7. Name of your present or most recent mathematics teacher:
   (CIRCLE ONE) Mr., Mrs., Miss

8. What is the approximate number of students in your present
   or most recent mathematics class? CHECK ONE

   Under 10   10-14   15-19   20-24   25-29   30-34   35-39   40 or more

9. How many times a week does your present (or most recent)
   mathematics class meet? (CHECK ONE)

   1 or 2   3 or 4   5 or 6   7 or 8

   9 or 10   11 or 12   13 or more

10. About how many hours a week does your present or most
    recent mathematics class meet? (CHECK ONE)

    Less than 1 hr.   1 hr. to 1 hr. 59 min.

    2 hrs. to 2 hrs. 59 min.   3 hrs. to 3 hrs. 59 min.

    4 hrs. to 4 hrs. 59 min.   5 hrs. to 5 hrs. 59 min.

    6 hrs. to 6 hrs. 59 min.   7 hrs. to 7 hrs. 59 min.

    8 hrs. or more

11. Do not write in these spaces
11. About how many hours do you attend school each week, not including lunch and recess breaks? (CHECK ONE)

<table>
<thead>
<tr>
<th>Less than 20</th>
<th>20-22</th>
<th>23-25</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>26-28</th>
<th>29-31</th>
<th>32-34</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>35-37</th>
<th>38-40</th>
<th>41 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12. About how many hours a week do you usually devote to
   a. Mathematics homework? __________________________
   b. All homework (including mathematics)? ____________

13. Where do your parents live now? (CHECK ONE)

   Farm

   Village-population less than 2,500

   Small city or town--population 2,500 - 15,000

   Medium sized city -- population 15,000 - 100,000

   Suburban area -- community near a large city (population of more than 100,000)

   Large city -- population of more than 100,000

14. What was the highest school grade that your father completed? (CIRCLE ONE)

<table>
<thead>
<tr>
<th>Elementary School</th>
<th>Junior and Sr. High</th>
<th>College</th>
<th>Graduate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5 6</td>
<td>7 8 9 10 11 12</td>
<td>13 14 15 16 17+</td>
<td></td>
</tr>
</tbody>
</table>

15. What was the highest school grade that your mother completed? (CIRCLE ONE)

<table>
<thead>
<tr>
<th>Elementary School</th>
<th>Junior and Sr. High</th>
<th>College</th>
<th>Graduate</th>
</tr>
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<tr>
<td>1 2 3 4 5 6</td>
<td>7 8 9 10 11 12</td>
<td>13 14 15 16 17+</td>
<td></td>
</tr>
</tbody>
</table>

16. Father's occupation
   If your father is not living, give his occupation before he died. If your father is not employed at present, give his usual occupation.
17. On the lines below, describe his occupation as carefully as you can. For example, if his occupation is "salesman," tell what he sells and where he works. If he works for a city government, be sure to indicate whether he is a policeman, clerk, etc. If he is a "plant manager," describe the kind of plant he manages, its size, location, and what it produces. If his occupation is "teacher," tell at what grade level, which subjects and where he teaches, etc.


18. Has your mother, at present, a paid job? (CHECK ONE)
   a) No ______
   b) Yes--full time ______
   c) Yes--part time ______

19. How many more years of school do you expect or plan to complete? (CHECK ONE)
   1 year or less _____ 2 years _____ 3 years _____
   4 years _____ 5 years _____ 6 years _____
   7 years _____ 8 years _____ 9 or more years _____

20. If you could do as you wished, how many more years of full-time education would you like to complete? (CHECK ONE)
   1 year or less _____ 2 years _____ 3 years _____
   4 years _____ 5 years _____ 6 years _____
   7 years _____ 8 years _____ 9 or more years _____

21. What occupation do you expect to enter after you have completed your schooling? ________________________________

22. If you could do as you wished, what occupation would you like to enter? ________________________________

23. Do you expect to take any more mathematics courses after this year? (CHECK ONE)
   Yes ______
   No ______
24. If you could do as you wished, would you like to take any more mathematics courses after this year? (CHECK ONE)
   Yes _____
   No _____

25. Have you been a member of any mathematics club or have you attended any special lectures on mathematics? (CHECK ONE)
   Yes _____
   No _____

   If the answer to the above question is "Yes," please describe briefly the club or lectures.

_________________________________________________________________
_________________________________________________________________
_________________________________________________________________
_________________________________________________________________

26. Which mathematics subject or subjects are you now studying or have recently studied?

_________________________________________________________________
_________________________________________________________________
_________________________________________________________________

27. Which two school subjects have you liked most? Write in names below.

_________________________________________________________________
_________________________________________________________________

28. Which two school subjects have you liked least? Write in names below.

_________________________________________________________________
_________________________________________________________________

29. In which two subjects have you generally gotten your highest marks? Write in names below.

_________________________________________________________________
_________________________________________________________________

30. In which two subjects have you generally gotten your lowest marks? Write in names below.

_________________________________________________________________
STUDENT QUESTIONNAIRE

International Educational Achievement Study
1. Name of your school

2. Name (PRINT)
   Last   First   Middle

3. Grade in school

4. Sex (CHECK ONE)  Boy  Girl

5. Date of birth:
   Month   Day   Year

6. CHECK ONE: Your present school, course or program is primarily
   a) academic
      1) specializing in mathematics
      2) studying mathematics but not specializing in it
      3) with no secondary mathematics
   b) vocational or commercial
      1) with secondary mathematics
      2) with no secondary mathematics
   c) general
      1) with secondary mathematics
      2) with no secondary mathematics

7. Name of your present or most recent mathematics teacher:
   (CIRCLE ONE) Mr., Mrs., Miss

8. What is the approximate number of students in your present or most recent mathematics class? CHECK ONE
   Under 10   10-14   15-19   20-24   
   25-29   30-34   35-39   40 or more
9. How many times a week does your present (or most recent) mathematics class meet? (CHECK ONE)
   1 or 2 ___  3 or 4 ___  5 or 6 ___  7 or 8 ___  9 or 10 ___  11 or 12 ___  13 or more ___

10. About how many hours a week does your present or most recent mathematics class meet? (CHECK ONE)
    Less than 1 hr. _____  1 hr. to 1 hr. 59 min. _____
    2 hrs. to 2 hrs. 59 min. _____  3 hrs. to 3 hrs. 59 min. _____
    4 hrs. to 4 hrs. 59 min. _____  5 hrs. to 5 hrs. 59 min. _____
    6 hrs. to 6 hrs. 59 min. _____  7 hrs. to 7 hrs. 59 min. _____
    8 hrs. or more _____

11. About how many hours do you attend school each week, not including lunch and recess breaks? (CHECK ONE)
    Less than 20 ___  20-22 ___  23-25 ___
    26-28 ___  29-31 ___  32-34 ___
    35-37 ___  38-40 ___  41 or more ___

12. About how many hours a week do you usually devote to
   a. Mathematics homework? ________________________
   12a ______
   b. All homework (including mathematics)? ____________
   12b ______

13. Where do your parents live now? (CHECK ONE)
    Farm ___
    Village-population less than 2,500 ___
    Small city or town -- population 2,500 - 15,000 ___
    Medium sized city -- population 15,000 - 100,000 ___
    Suburban area -- community near a large city (population of more than 100,000) ___
    Large city -- population of more than 100,000 ___

14. What was the highest school grade that your father completed? (CIRCLE ONE)
   Elementary School  Junior and Sr. High  College  Graduate
   1 2 3 4 5 6  7 8 9  10 11 12  13 14 15 16  17+
15. What was the highest school grade that your mother completed? (CIRCLE ONE)

<table>
<thead>
<tr>
<th>Elementary School</th>
<th>Junior and Sr. High</th>
<th>College</th>
<th>Graduate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5 6</td>
<td>7 8 9</td>
<td>10 11 12</td>
<td>13 14 15 16 17+</td>
</tr>
</tbody>
</table>

16. Father's occupation

If your father is not living, give his occupation before he died. If your father is not employed at present, give his usual occupation.

17. On the lines below, describe his occupation as carefully as you can. For example, if his occupation is "salesman," tell what he sells and where he works. If he works for a city government, be sure to indicate whether he is a policeman, clerk, etc. If he is a "plant manager," describe the kind of plant he manages, its size, location, and what it produces. If his occupation is "teacher," tell at what grade level, which subjects and where he teaches, etc.

18. Has your mother, at present, a paid job? (CHECK ONE)
  a) No ________
  b) Yes--full time ________
  c) Yes--part time ________

19. How many more years of school do you expect or plan to complete? (CHECK ONE)

<table>
<thead>
<tr>
<th>1 year or less</th>
<th>2 years</th>
<th>3 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 years</td>
<td>5 years</td>
<td>6 years</td>
</tr>
<tr>
<td>7 years</td>
<td>8 years</td>
<td>9 or more years</td>
</tr>
</tbody>
</table>

20. If you could do as you wished, how many more years of full-time education would you like to complete? (CHECK ONE)

<table>
<thead>
<tr>
<th>1 year or less</th>
<th>2 years</th>
<th>3 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 years</td>
<td>5 years</td>
<td>6 years</td>
</tr>
<tr>
<td>7 years</td>
<td>8 years</td>
<td>9 or more years</td>
</tr>
</tbody>
</table>
21. What occupation do you expect to enter after you have completed your schooling? ________________________

22. If you could do as you wished, what occupation would you like to enter? ________________________

23. Do you expect to take any more mathematics courses after this year? (CHECK ONE)
   Yes ___  No ___

24. If you could do as you wished, would you like to take any more mathematics courses after this year? (CHECK ONE)
   Yes ___  No ___

25. Have you been a member of any mathematics club or have you attended any special lectures on mathematics? (CHECK ONE)
   Yes ___  No ___

   If the answer to the above question is "Yes," please describe briefly the club or lectures.

   ____________________________________________
   ____________________________________________
   ____________________________________________

26. When you leave school, which of the following do you intend to do? (CHECK ONE)

   a) Attend a university or college full-time ________
   b) Attend a university or college part-time ________
   c) Attend some other institution of higher learning full-time ________
   d) Attend some other institution of higher learning part-time ________
   e) Work full-time ________
   f) Do something different (Please specify) ________

27. Have you taken or are you planning to take any state, regional or national examination in mathematics? (CHECK ONE)
   Yes __________________
   No __________________
   Undecided ____________
Here is a list of secondary school subjects. Use the numbers in this list to answer questions 28 - 32 on the following page.

<table>
<thead>
<tr>
<th>No.</th>
<th>Subject</th>
<th>No.</th>
<th>Subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>English (Language Usage)</td>
<td>72</td>
<td>Advanced Algebra</td>
</tr>
<tr>
<td>02</td>
<td>Speech and Dramatics</td>
<td>73</td>
<td>College Algebra</td>
</tr>
<tr>
<td>03</td>
<td>Literature</td>
<td>79</td>
<td>Experimental Algebra Course</td>
</tr>
<tr>
<td>10</td>
<td>Foreign language</td>
<td>80</td>
<td>Plane Geometry</td>
</tr>
<tr>
<td>20</td>
<td>Social Studies</td>
<td>85</td>
<td>Trigonometry</td>
</tr>
<tr>
<td>30</td>
<td>General Science</td>
<td>89</td>
<td>Experimental Geometry or Trigonometry</td>
</tr>
<tr>
<td>31</td>
<td>Biological Science</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>Physical Science</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>Art</td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>Music</td>
<td>90</td>
<td>Calculus</td>
</tr>
<tr>
<td>52</td>
<td>Agriculture</td>
<td>91</td>
<td>Advanced Mathematics</td>
</tr>
<tr>
<td>53</td>
<td>Shop</td>
<td></td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>Home Economics</td>
<td></td>
<td>(specify)</td>
</tr>
<tr>
<td>55</td>
<td>Commercial subjects</td>
<td>99</td>
<td>Experimental Advanced Mathematics</td>
</tr>
<tr>
<td>56</td>
<td>Physical Education</td>
<td></td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>Mechanical Drawing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>Arithmetic only</td>
<td></td>
<td>(specify)</td>
</tr>
<tr>
<td>61</td>
<td>General Mathematics</td>
<td>101</td>
<td>Other</td>
</tr>
<tr>
<td>69</td>
<td>Experimental Arithmetic or General Mathematics</td>
<td></td>
<td>(specify)</td>
</tr>
<tr>
<td></td>
<td>Program</td>
<td>102</td>
<td>Other</td>
</tr>
<tr>
<td>70</td>
<td>Elementary Algebra</td>
<td>103</td>
<td>Other</td>
</tr>
<tr>
<td>71</td>
<td>Intermediate Algebra</td>
<td></td>
<td>(specify)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>104</td>
<td>Other</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(specify)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>105</td>
<td>Other</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(specify)</td>
</tr>
</tbody>
</table>
28. Which two secondary school subjects have you liked most? Select numbers from the preceding list and write them on the lines below.

__________________________________________

__________________________________________

29. Which two secondary school subjects have you liked least? Select numbers from the preceding list and write them on the lines below.

__________________________________________

__________________________________________

30. In which two subjects did you get your highest grades? Select numbers from the preceding list and write them on the lines below.

__________________________________________

__________________________________________

31. In which two subjects did you get your lowest grades? Select numbers from the preceding list and write them on the lines below.

__________________________________________

__________________________________________

32. What mathematics courses did you take in each of the following grades? Insert the numbers of the courses from the preceding list on each line below. If you did not take any mathematics courses in a given year, leave that line blank.

9th  

10th  

11th  

12th  

32  _______
STUDENT QUESTIONNAIRE

International Educational Achievement Study
1. Name of your school ____________________________

2. Name (PRINT) ________________________________
   Last   First   Middle

3. Grade in school ________________

4. Sex (CHECK ONE) Boy _____ Girl _____

5. Date of birth: ____________________________
   Month   Day   Year

6. CHECK ONE: Your present school, course or program is primarily

   a) academic
      1) specializing in mathematics
      2) studying mathematics but not specializing in it
      3) with no secondary mathematics

   b) vocational or commercial
      1) with secondary mathematics
      2) with no secondary mathematics

   c) general
      1) with secondary mathematics
      2) with no secondary mathematics

7. Name of your present or most recent mathematics teacher:

   (CIRCLE ONE) Mr., Mrs., Miss ____________________________

8. What is the approximate number of students in your present or most recent mathematics class? CHECK ONE

   Under 10 _____ 10-14 _____ 15-19 _____ 20-24 _____
   25-29 _____ 30-34 _____ 35-39 _____ 40 or more _____
9. How many times a week does your present (or most recent) mathematics class meet? (CHECK ONE)

1 or 2 _____ 3 or 4 _____ 5 or 6 _____ 7 or 8 _____
9 or 10_____ 11 or 12_____ 13 or more _____

10. About how many hours a week does your present or most recent mathematics class meet? (CHECK ONE)

Less than 1 hr. _____ 1 hr. to 1 hr. 59 min. _____
2 hrs. to 2 hrs. 59 min. _____ 3 hrs. to 3 hrs. 59 min. _____
4 hrs. to 4 hrs. 59 min. _____ 5 hrs. to 5 hrs. 59 min. _____
6 hrs. to 6 hrs. 59 min. _____ 7 hrs. to 7 hrs. 59 min. _____
8 hrs. or more _____

11. About how many hours do you attend school each week, not including lunch and recess? (CHECK ONE)

Less than 20 _____ 20-22 _____ 23-25 _____
26-28 _____ 29-31 _____ 32-34 _____
35-37 _____ 38-40 _____ 41 or more _____

12. About how many hours a week do you usually devote to

a. Mathematics homework? _______________________

b. All homework (including mathematics)? ________

13. Where do your parents live now? (CHECK ONE)

Farm
Village — population less than 2,500 ________
Small city or town — population 2,500 - 15,000 ________
Medium sized city — population 15,000 - 100,000 ________
Suburban area — community near a large city (population of more than 100,000) ________
Large city — population of more than 100,000 ________

14. What was the highest school grade that your father completed? (CIRCLE ONE)

Elementary School  Junior and Sr. High  College  Graduate
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17+
15. What was the highest school grade that your **mother** completed? (CIRCLE ONE)

<table>
<thead>
<tr>
<th>Elementary School</th>
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</table>

16. **Father's occupation**
   If your father is not living, give his occupation before he died. If your father is not employed at present, give his usual occupation.

17. On the lines below, describe his occupation as carefully as you can. For example, if his occupation is "salesman," tell what he sells and where he works. If he works for a city government, be sure to indicate whether he is a policeman, clerk, etc. If he is a "plant manager," describe the kind of plant he manages, its size, location, and what it produces. If his occupation is "teacher," tell at what grade level, which subjects and where he teaches, etc.

18. Has your mother, at present, a paid job? (CHECK ONE)
   a) No
   b) Yes--full time
   c) Yes--part time

19. How many more years of school do you **expect** or plan to complete? (CHECK ONE)

<table>
<thead>
<tr>
<th>1 year or less</th>
<th>2 years</th>
<th>3 years</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>7 years</td>
<td>8 years</td>
<td>9 or more years</td>
</tr>
</tbody>
</table>

20. If you could do as you wished, how many more years of full-time education would you **like** to complete? (CHECK ONE)

<table>
<thead>
<tr>
<th>1 year or less</th>
<th>2 years</th>
<th>3 years</th>
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</tr>
<tr>
<td>7 years</td>
<td>8 years</td>
<td>9 or more years</td>
</tr>
</tbody>
</table>
21. What occupation do you expect to enter after you have completed your schooling? _____________________________ 21 ______

22. If you could do as you wished, what occupation would you like to enter? _____________________________ 22 ______

23. Do you expect to take any more mathematics courses after this year? (CHECK ONE)

Yes ___ No ___ 23 ______

24. If you could do as you wished, would you like to take any more mathematics courses after this year? (CHECK ONE)

Yes ___ No ___ 24 ______

25. Have you been a member of any mathematics club or have you attended any special lectures on mathematics? (CHECK ONE)

Yes ___ No ___ 25 ______

If the answer to the above question is "Yes," please describe briefly the club or lectures.

__________________________________________________________________________________________

__________________________________________________________________________________________

__________________________________________________________________________________________

26. When you leave school, which of the following do you intend to do? (CHECK ONE)

a) Attend a university or college full-time ____________

b) Attend a university or college part-time ____________

c) Attend some other institution of higher learning full-time ____________

d) Attend some other institution of higher learning part-time ____________

e) Work full-time ____________

f) Do something different (Please specify) ____________ 26 ______

27. Have you taken or are you planning to take any state, regional or national examination in mathematics? (CHECK ONE)

Yes ____________________

No ____________________

Undecided ________________ 27 ______
Here is a list of secondary school subjects. Use the numbers in this list to answer questions 28 - 32 on the following page.

01 English (Language Usage) 72 Advanced Algebra
02 Speech and Dramatics 73 College Algebra
03 Literature 79 Experimental Algebra Course
10 Foreign language 80 Plane Geometry
20 Social Studies 85 Trigonometry
30 General Science 89 Experimental Geometry or Trigonometry
31 Biological Science 89 Experimental Geometry or Trigonometry
40 Physical Science Course
50 Art
51 Music 90 Calculus
52 Agriculture 91 Advanced Mathematics
53 Shop
54 Home Economics (specify) ______
55 Commercial subjects 99 Experimental Advanced
56 Physical Education Mathematics
57 Mechanical Drawing
60 Arithmetic only (specify) ______
61 General Mathematics 101 Other
69 Experimental Arithmetic or (specify) ______
   General Mathematics 102 Other
   Program (specify) ______
70 Elementary Algebra 103 Other
71 Intermediate Algebra (specify) ______
104 Other
   (specify) ______
105 Other (specify) ______
28. Which two secondary school subjects have you liked most? Select numbers from the preceding list and write them on the lines below.

__________________________________________

__________________________________________

29. Which two secondary school subjects have you liked least? Select numbers from the preceding list and write them on the lines below.

__________________________________________

__________________________________________

30. In which two subjects did you get your highest grades? Select numbers from the preceding list and write them on the lines below.

__________________________________________

__________________________________________

31. In which two subjects did you get your lowest grades? Select numbers from the preceding list and write them on the lines below.

__________________________________________

__________________________________________

32. What mathematics courses did you take in each of the following grades? Insert the numbers of the courses from the preceding list on each line below. If you did not take any mathematics courses in a given year, leave that line blank.

9th ______________________________________

10th ______________________________________

11th ______________________________________

12th ______________________________________
SCHOOL QUESTIONNAIRE

International Educational Achievement Study

SCHOOL TYPE ________________________________ COUNTRY __________________________

GROUPS TESTED: Circle groups that apply: LEVEL ____________________________
13 Yrs.+ Gr.8 Gr.10 Gr.12Math. Gr.12Non-Math ____________________________

SCHOOL NAME ____________________________ SCHOOL __________________________

Where alternative responses to the questions below are given a number, please circle the appropriate number or numbers and leave the others unmarked.

Example: Yes . . . . . . . 1 indicates "Yes"
No . . . . . . . 2

1. What is the total enrollment of your school? Give the number as accurately as you can. ____________________________

2. What is the total full-time (2 part-time are equal to 1 full-time) teaching staff in your school? Exclude administrative staff.

   Less than 10 . . . . . . . 1
   11-20 . . . . . . . . . . . 2
   21-30 . . . . . . . . . . . 3
   31-40 . . . . . . . . . . . 4
   41-50 . . . . . . . . . . . 5
   51-60 . . . . . . . . . . . 6
   61-70 . . . . . . . . . . . 7
   71-80 . . . . . . . . . . . 8
   81 or more . . . . . . . . 9

3. How many members of your staff teach mathematics?

   1 2 3 4 5 6 7 8 9 or more

4. How many of these have specialist training in mathematics?

   0 1 2 3 4 5 6 7 8 or more

5. What percent of all those who teach mathematics are men?

   0% . . . . . . . . . . . 1
   1-25% . . . . . . . . . . 2
   26-50% . . . . . . . . . . 3
   51-75% . . . . . . . . . . 4
   76-99% . . . . . . . . . . 5
   100% . . . . . . . . . . . 6
6. Is your school
   a boys' school ........................................ 1
   a girls' school ........................................ 2
   co-educational with boys and girls
   taught mathematics together .......................... 3
   co-educational but boys and girls
   taught mathematics separately ........................ 4

7. Which of the following best describes your school?

   Comprehensive, offering appropriate courses for
   students of all ranges of ability ..................... 1
   Selective-academic:
   Limited to those with high academic ability .......... 2
   Selective vocational:
   Limited to those with special ability for
   some vocational field ................................... 3
   Offering courses to the remainder of students
   after selection ........................................... 4

Comment


8. To what extent does ability grouping take place within your
   school?

   It is practiced for all pupils .......................... 1
   It is practiced for some pupils at all levels .......... 2
   It is practiced in some age or grade groups only ..... 3
   (Indicate in which groups under "Comment")
   It is not practiced at all ................................ 4

Comment


9. Please indicate the average number of different subjects (in which
   grades are given) taken by a student in your school at the following
   levels. Please circle the appropriate number in each row.

<table>
<thead>
<tr>
<th>Average number of subjects taken</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 12 ................................. 1 2 3 4 5 6 7 8 9 or more</td>
</tr>
<tr>
<td>Grade 11 ................................. 1 2 3 4 5 6 7 8 9 or more</td>
</tr>
<tr>
<td>Grade 10 ................................. 1 2 3 4 5 6 7 8 9 or more</td>
</tr>
<tr>
<td>Grade 9 ................................. 1 2 3 4 5 6 7 8 9 or more</td>
</tr>
<tr>
<td>Grade 8 ................................. 1 2 3 4 5 6 7 8 9 or more</td>
</tr>
</tbody>
</table>
10. What is the total annual budget in your school covering the salaries of all professional staff? (Include teachers and all other professional workers such as librarians, psychologists, etc. Do not include secretaries, caretakers, janitors, etc.)

11. What is the total annual recurrent expenditure for equipment in your school? (Capital equipment is not to be included).

12. Does your school provide special academic opportunities in mathematics for the most able students?

   Yes . . . . . . . . . . 1
   No . . . . . . . . . . 2

13. a) Is your school involved in an experimental program in the teaching of mathematics?

   Yes . . . . . . . . . . 1
   No . . . . . . . . . . 2

   b) If yes, describe briefly.


14. How many hours (60 minutes) instruction per year do students in your school receive?

   800 or less . . . . . . 1
   801 - 900 . . . . . . 2
   901 - 1,000 . . . . . . 3
   1,001 - 1,100 . . . . . . 4
   1,101 - 1,200 . . . . . . 5
   1,201 - 1,300 . . . . . . 6
   1,301 - 1,400 . . . . . . 7
   1,401 - 1,500 . . . . . . 8
   more than 1,500 . . . . . 9

15. What is the grade range in your school?

   from Grade . . . . . . to Grade . . . .

   Thank you for your cooperation!
TEACHER QUESTIONNAIRE

International Educational Achievement Study

GROUPS TESTED: Circle groups that apply:
13 Yrs.+ Gr.8 Gr.10 Gr.12 Math. Gr.12-Non-Math

SCHOOL NAME ________________________________

Where alternative responses to the questions below are indicated by a number, please circle the appropriate number, and leave the others unmarked.

Example: Yes ........ 1 indicates "Yes"
No ............ 2

1. Name ________________________________

2. Age
   18 - 23 ............ 1
   24 - 29 ............ 2
   30 - 39 ............ 3
   40 - 49 ............ 4
   50 - 59 ............ 5
   60 + ............ 6

3. Sex
   M ............ 1
   F ............ 2

4. Please indicate the total post-secondary school training which you received in preparation for your present assignment. Include both professional (teaching methods, educational psychology, etc.) and the subject matter of mathematics.

Length of total training in years
   1 2 3 4 5 or more

5. In what type of higher educational institution did you receive your professional training?
   University or liberal arts college ............ 1
   Teacher training college ............ 2
   Both of the above ............ 3
   Other ............ 4
   None ............ 5

6. In what type of educational institution did you receive your mathematics training?
   University or liberal arts college ............ 1
   Teacher training college ............ 2
   Both of the above ............ 3
   Other ............ 4
   None ............ 5
7. a) Have you completed your formal training for state certification?

Yes .................................. 1
No. ...................................... 2

b) Are you certified specifically for teaching mathematics?

Yes .................................. 1
No. ...................................... 2

8. How many years of teaching experience have you had?

Less than 1 year ....................... 1
1 - 2 years .............................. 2
3 - 5 years .............................. 3
6 - 10 " ................................. 4
11 - 15 " ................................. 5
16 - 20 " ................................. 6
21 - 25 " ................................. 7
26+ ...................................... 8

9. What amount of in-service training programs in mathematics have you had during the past 5 years?

No in-service training .................. 1
A total of 1 - 3 weeks .................. 2
" " 4 - 6 " ................................. 3
" " 7 - 12 " ................................. 4
" more than 12 weeks .................. 5

10. How many lectures and/or short seminars on the teaching of mathematics have you attended during the past 2 years?
(not including entries in 9)

0 ........................................... 1
1 - 3 ....................................... 2
4 - 6 ....................................... 3
7 - 9 ....................................... 4
10+ ......................................... 5

11. If you have indicated a positive response to either 9 or 10, have you attended any lectures, seminars or courses dealing with "New Mathematics?"

Yes .................................... 1
No. ...................................... 2

12. a) Are you at present basing any of your teaching on the "New Mathematics?"

Yes .................................... 1
No. ...................................... 2
b) If yes, describe briefly ________________________________


13. Circle only one of the following three statements:

In teaching mathematics, I consider the logical structure of mathematics to be of prime importance ... 1

In teaching mathematics, I consider the interest and needs of the students to be of prime importance ... 2

I am uncertain which of the two preceding statements I would endorse ... 3

14. Do you consider that having to use a set or required syllabus places a restriction on the way in which you teach mathematics?

Yes ... 1
No ... 2
Does not apply ... 3

15. Do you consider that having to use set or required textbooks places a restriction on the way in which you teach mathematics?

Yes ... 1
No ... 2
Does not apply ... 3

16. Do you consider that having to employ set methods of teaching places a restriction on the way in which you teach mathematics?

Yes ... 1
No ... 2
Does not apply ... 3

17. Do you consider that end of course examinations place a restriction on the way in which you teach mathematics?

Yes ... 1
No ... 2
Does not apply ... 3

18. Attached to this questionnaire are the sets of tests that are being given to some of your students. Also attached is the special answer sheet on which they are recording their answers by blackening the appropriate response position. The response positions for the questions in each test are located in blocks on the answer sheets, e.g.: T - A, T - 3, T - 6.
To have information available concerning the appropriateness of each item for your students, you are now asked to rate the questions as to whether or not the topic any particular question deals with, has been covered by the students to whom you teach mathematics and who are taking this set of tests. Even if you are not sure, please make an estimate according to the scale given below.

Please examine each question in turn and indicate in the way described below whether, in your opinion

- **A.** All or most (at least 75%) of this group of students have had an opportunity to learn this type of problem.

- **B.** Some (25-75%) of this group of students have had an opportunity to learn this type of problem.

- **C.** Few or none (under 25%) of this group of students have had an opportunity to learn this type of problem.

Please indicate your rating (A, B, or C) by blackening the appropriate response position for each question on the attached answer sheet.

Before carrying out this procedure, would you please complete the back of the answer sheet by filling in your name on line 1, the name of the school in which you teach on line 2, and on line 3 the grades containing the students whom you teach and who are taking this set of tests.

If you teach groups of students taking different sets of tests, will you please complete the above rating procedure separately for each of the groups -- that is, rate the questions in the tests taken by the one group of students, then rate the questions in the tests taken by the second group. The appropriate tests and answer sheets are attached separately to this questionnaire.

Thank you for your cooperation!
STUDENT OPINION BOOKLET

The purpose of this STUDENT OPINION BOOKLET is to find out how students think about a number of things. You should give answers to these statements as rapidly and as accurately as you can. If you are not taking a mathematics course now, answer the statements about mathematics on the basis of your most recent mathematics course. There are no "right" or "wrong" answers for these statements, so please answer according to your own opinions. Your answers will have no effect on your school marks.

Directions

Your answers to these statements should be marked in the section of your answer sheet labelled SO (Student Opinion). The numbers in this section of the answer sheet correspond to the numbered statements in this booklet. In answering statements 1-22, read each statement carefully and mark:

Answer space A - if the statement is usually TRUE about your school.

Answer space B - if the statement is usually NOT TRUE about your school.

Answer space C - if you cannot decide whether the statement is usually TRUE or NOT TRUE about your school.

1. Most schoolwork is the memorizing of information.
2. In our school we get a great deal of practice and drill until we are almost perfect in our learning.
3. The students spend most of their class time listening to the teachers and taking notes.
4. My mathematics teacher shows us different ways of solving the same problem.
5. Our teachers want us to do most of our learning from the textbook which is used in the course.
6. My mathematics teacher does not like students to ask questions after he has given an explanation.

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7. My mathematics teacher wants students to solve problems only by the procedures he teaches.

8. We are expected to learn and discover many ideas for ourselves.

9. We are expected to develop a thorough understanding of ideas and not to just memorize information.

10. Our teachers believe in strict discipline and each student does exactly what he is told to do.

11. Students are encouraged to devise their own projects or experiments in order to learn on their own.

12. My mathematics teacher expects us to learn how to solve problems by ourselves, but helps us when we have difficulties.

13. In my mathematics class, students who have original ideas get better grades than do students who are most careful and neat in their work.

14. Most of our classroom work is listening to the teacher.

15. My mathematics teacher requires the students not only to master the steps in solving problems, but also to understand the reasoning involved.

16. My mathematics teacher encourages us to try to find several different methods for solving particular problems.

17. My mathematics course requires more thinking about the methods of solving problems than memorization of rules and formulae.

18. My mathematics teacher wants us to discover mathematical principles and ideas for ourselves.

19. My mathematics teacher explains the basic ideas; we are expected to develop the methods of solution for ourselves.

20. We do not use just one textbook for most of our subjects. Various sources and books from which we can learn are suggested to us.

21. Most of the problems my mathematics teacher assigns are to give us practice in using a particular rule or formula.

22. Much of our classroom work is discussing ideas and problems with the teacher and the other students.
Directions

In answering statements 23-65, read each statement carefully and mark:

Answer space A - if you AGREE with the statement.

Answer space B - if you DISAGREE with the statement.

Answer space C - if you cannot decide whether you AGREE or DISAGREE with the statement

23. In mathematics there is always a rule to follow in solving problems.
24. I generally like my school work.
25. It should be possible to eliminate war once and for all.
26. Success depends to a large part on luck and fate.
27. More of the most able people should be encouraged to become mathematicians and mathematics teachers.
28. Some day most of the mysteries of the world will be revealed by science.
29. Anyone can learn mathematics.
30. Most school learning has little value for a person.
31. By improving industrial and agricultural methods, poverty can be eliminated in the world.
32. I dislike school and will quit just as soon as possible.
33. With increased medical knowledge, it should be possible to lengthen the average life span to 100 years or more.
34. Outside of science and engineering, there is little need for mathematics (Algebra, Geometry, etc.) in most jobs.
35. Mathematics is of great importance to a country's development.
36. The most important reason for studying arithmetic and secondary school mathematics is that they help people to take care of their own financial affairs.
37. Very few people can learn mathematics.
38. Mathematics helps one to think according to strict rules.
39. Mathematics (Algebra, Geometry, etc.) is not useful for the problems of everyday life.
40. Someday the deserts will be converted into good farming land by the application of engineering and science.
41. I am bored most of the time in school.
42. Almost all of present day mathematics was known at least a century ago.
43. Education can only help people develop their natural abilities; it cannot change people in any fundamental way.

44. I enjoy everything about school.

45. A thorough knowledge of advanced mathematics is the key to an understanding of our world in the 20th century.

46. School is not very enjoyable, but I can see value in getting a good education.

47. It is important to know mathematics (Algebra, Geometry, etc.) in order to get a good job.

48. Almost anyone can learn mathematics if he is willing to study.

49. Mathematics is a very good field for creative people to enter.

50. Unless one is planning to become a mathematician or scientist, the study of advanced mathematics is not very important.

51. Any person of average intelligence can learn to understand a good deal of mathematics.

52. The most enjoyable part of my life is the time I spend in school.

53. Even complex mathematics can be made understandable and useful to every high school student.

54. In the near future most jobs will require a knowledge of advanced mathematics.

55. With hard work anyone can succeed.

56. Almost every present human problem will be solved in the future.

57. Almost all students can learn complex mathematics if it is properly taught.

58. I like all school subjects.

59. There is little place for originality in mathematics.

60. I enjoy most of my school work and want to get as much additional education as possible.

61. Only people with a very special talent can learn mathematics.

62. Mathematics will change rapidly in the near future.

63. Although school is difficult, I want as much education as I can get.

64. In the study of mathematics, if the student misses a few lessons it is difficult to catch up.

65. I find school interesting and challenging.
International Mathematics Test

Test A

Test Directions:

This short test samples many different topics in mathematics. Some problems you will probably know how to do, and some you will probably not know how to do. Do the very best you can, but do not waste time puzzling over any one question. If you get stuck on a question, skip it and go on to the next one. If you finish the test before the time is up, you can go back and work on any questions that you have skipped.

Do not make any marks on the test booklet. Use the separate scratch paper for your figuring. Be sure to mark the correct row on your answer sheet for each question.

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(It is forbidden to reproduce this material in any way.)
1. 43.0 – 17.6 is equal to

2. How many seven-man teams can you make out of 7 nine-man teams?
   A. 7   B. 8   C. 9   D. 16   E. 63

3. (22 × 18) – (47 + 59) is equal to
   A. 290   B. 300   C. 384   D. 408   E. 502

4. In the figure shown below the little squares are all the same size and the area of the whole rectangle is equal to 1.

   The area of the shaded part is equal to
   A. \( \frac{2}{15} \)   B. \( \frac{1}{3} \)   C. \( \frac{2}{5} \)   D. \( \frac{3}{8} \)   E. \( \frac{1}{2} \)

5. In the graph on the right, rainfall in inches is plotted for 13 weeks. The average weekly rainfall during the period is approximately
   A. 1 inch   B. 2 inches   C. 3 inches   D. 4 inches   E. 5 inches

6. The value of \( 2^3 \times 3^2 \) is
   A. 30   B. 36   C. 64   D. 72   E. none of these
7. A box has a volume of 100 cc. Another box is twice as long, twice as wide and twice as high. How many cc. is the volume of the second box?

8. There is a brass plate of the shape and dimensions shown in the adjoining figure. What is its area in square inches?

   ![Diagram of a brass plate with dimensions 4'' x 90° x 90° x 8'']

   A. 16 D. 64
   B. 24 E. 96
   C. 32

9. What is the square root of $12 \times 75$?

   A. 6.25 B. 30 C. 87 D. 625 E. 900

10. Three straight lines intersect as shown in the figure on the right. What is $x$ equal to in degrees?

    ![Diagram of three intersecting lines with angles 80° and 150°]

    A. 30 D. 110
    B. 50 E. 150
    C. 60

11. A shopkeeper has $x$ lb. of tea in stock. He sells 15 lb. and then receives a new lot weighing $2y$ lb. What weight of tea does he now have?

    A. $x - 15 - 2y$ D. $x + 15 - 2y$
    B. $x + 15 + 2y$ E. none of the above
    C. $x - 15 + 2y$

12. If $\frac{x}{2} < 7$, then

    A. $x < \frac{7}{2}$ B. $x < 5$ C. $x < 14$ D. $x > 5$ E. $x > 14$
13. A piece of tin with dimensions as shown is to be folded along the dotted lines to make a box. What is the volume, in cubic centimeters, enclosed in the box?

\[5\text{cm.} \quad 4\text{cm.} \quad 7\text{cm.}\]

14. If \( \frac{4x}{12} = 0 \), then \( x \) is equal to

A. 0  
B. 3  
C. 8  
D. 12  
E. 16

15. The floor of a room is covered with wooden rectangular blocks. When blocks measuring \( a \) inches by \( b \) inches are used, \( M \) blocks are needed. If blocks fit exactly, how many blocks will be needed if each block measures \( x \) inches by \( y \) inches?

\[ \frac{Mab}{xy} \quad \frac{ab}{Mxy} \quad \frac{(a + b)M}{x + y} \quad \frac{ab \cdot xy}{M} \quad \frac{Mxy}{ab} \]

16. Which of the following sets of conditions is not sufficient for the congruence of \( \triangle FGH \) and \( \triangle PQR \) on the right when \( f \) is less than \( g \)?

A. \( \angle F = \angle P \)
   \( g = q \)
   \( f = p \)

B. \( \angle F = \angle P \)
   \( h = r \)
   \( \angle G = \angle Q \)

C. \( g = q \)
   \( \angle F = \angle P \)
   \( h = r \)

D. \( h = r \)
   \( g = q \)
   \( f = p \)

E. \( f = p \)
   \( \angle G = \angle Q \)
   \( h = r \)
17. Which of the following is (are) true?
   I. \((53 \times 73) \times 17 = 53 \times (73 \times 17)\)
   II. \(133 \times (78 + 89) = (133 \times 78) + 89\)
   III. \(133 \times (78 + 89) = (133 \times 78) + (133 \times 89)\)
   A. I only  
   B. II only  
   C. III only  
   D. I and II only  
   E. I and III only

18. There are 227 boys in a school. Every boy in the school belongs to either the music club or the sports club, and some boys belong to both clubs. The music club has 120 members, and 36 of these are also members of the sports club. What is the total membership of the sports club?

19. The lengths of the sides of a triangle \(XYZ\) are 4, 7, and 10. If a similar triangle has a perimeter of 147, what is the length of its shortest side?

20. In the solution of the following system of equations,

   \[
   \begin{align*}
   2x + y &= 7 \\
   x - 4y &= 4
   \end{align*}
   \]

   the value of \(y\) is equal to

   A. \(-\frac{5}{3}\)  
   B. \(-9\)  
   C. \(\frac{1}{9}\)  
   D. \(-\frac{1}{9}\)  
   E. \(\frac{5}{3}\)

21. Which of the following is true for any parallelogram \(ABCD\) which has an acute angle at \(B\) and diagonals \(AC\) and \(BD\)?

   A. \(AB < BC\)  
   B. \(AB = BC\)  
   C. \(AB > BC\)  
   D. \(AC < BD\)  
   E. None of them

22. The distance between two towns, \(A\) and \(B\), is 150 kilometers. This distance is represented on a certain map by a length of 30 centimeters. The scale of this map is

   A. \(1/500,000\)  
   B. \(30/150\)  
   C. \(1/20,000\)  
   D. \(1/5,000\)  
   E. \(1/200,000\)
23. Which of the following equals \( 7 \times (3 + 9) \)?

A. \((7 \times 3) + (7 \times 9)\)
B. \((7 \times 9) + (3 \times 9)\)
C. \((7 \times 3) + (3 \times 9)\)
D. \(7 \times 27\)
E. \(21 + 9\)
International Mathematics Test

Test B

Test Directions:

This short test samples many different topics in mathematics. Some problems you will probably know how to do, and some you will probably not know how to do. Do the very best you can, but do not waste time puzzling over any one question. If you get stuck on a question, skip it and go on to the next one. If you finish the test before the time is up, you can go back and work on any questions that you have skipped.

Do not make any marks on the test booklet. Use the separate scratch paper for your figuring. Be sure to mark the correct row on your answer sheet for each question.
1. \( \frac{2}{5} + \frac{3}{8} \) is equal to

A. \( \frac{5}{13} \)  B. \( \frac{5}{40} \)  C. \( \frac{6}{40} \)  D. \( \frac{16}{15} \)  E. \( \frac{31}{40} \)

2. Peter and Paul decided to start saving money. Peter can save 3 dollars each month and Paul can save 5 dollars. At this rate, after how many months will Paul have exactly 10 dollars more than Peter?

A. 2  B. 3  C. 4  D. 5  E. 8

3. In the division on the right, the correct answer is \( .004 \div 24.56 \)

A. .614  B. 6.14  C. 61.4  D. 614  E. 6140

4. The arithmetic mean (average) of: 1.50, 2.40, 3.75 is equal to

A. 2.40  B. 2.55  C. 3.75  D. 7.65  E. none of these

5. Which of the following operations with whole numbers will always give a whole number?

I. Addition  
II. Multiplication  
III. Division

A. I only  
B. II only  
C. III only  
D. I and II only  
E. II and III only

6. If the Selling Price of an article was $55 and a profit of 10% was made on the Cost Price, what was the cost price in dollars?

7. The value of \( 0.2131 \times 0.02958 \) is approximately

A. 0.6  B. 0.06  C. 0.006  D. 0.0006  D. 0.00006
8. Joe had three test scores of 78, 76, and 74, while Mary had scores of 72, 82, and 74. How did Joe's average compare with Mary's?

A. Joe's was 1 point higher.
B. Joe's was 1 point lower.
C. Both averages were the same.
D. Joe's was 2 points higher.
E. Joe's was 2 points lower.

9. Which of the following is false when \( a \) and \( b \) are different real numbers:

A. \( (a + b) + c = a + (b + c) \)
B. \( ab = ba \)
C. \( a + b = b + a \)
D. \( (ab)c = a(bc) \)
E. \( a - b = b - a \)

10. If \( P = LW \) and if \( P = 12 \) and \( L = 3 \), then \( W \) is equal to

A. \( \frac{3}{4} \)  
B. 3  
C. 4  
D. 12  
E. 36

11. Simplify: \( 5x + 3y + 2x - 4y \)

A. \( 7x + 7y \)  
B. \( 8x - 2y \)  
C. \( 6xy \)  
D. \( 7x - y \)  
E. \( 7x + y \)

12. What is the value of \( (-6) - (-8) \) ?

13. If \( AB \) is a straight line, what is the measure in degrees of angle \( BCD \) in the figure on the right?

A. 20  
B. 40  
C. 50  
D. 80  
E. 100
14. If $x = y = z = 1$, then $\frac{x - z}{x + y}$ is equal to

A. -2 B. -1 C. 0 D. $\frac{1}{2}$ E. 1

15. If $x = -3$, the value of $-3x$ is

A. -9 B. -6 C. -1 D. 1 E. 9

Use the graph below in answering the two following questions.

16. Three hours after starting, car A is how many miles ahead of car B?

A. 2 B. 10 C. 15 D. 20 E. 25

17. How much longer does it take car B to go 50 miles than it does for car A to go 50 miles?

A. 1 hour 15 minutes D. 2 hours 30 minutes
B. 1 hour 30 minutes
C. 2 hours E. 2 hours 45 minutes

18. In $\triangle KLM$ on the right, $KL = KM$, $PO \perp LM$, and LP is a straight line. Then $\triangle NKP$ is isosceles because

A. $\angle P = \angle KNP$, since both are complements of the equal angles L and M.
B. $NK = PK$, since $\angle P = \angle M$.
C. its sides are parallel to the sides of $\triangle KLM$.
D. its sides are perpendicular to the sides of $\triangle KLM$.
E. $\angle P = \angle KNP$, since both are half the supplement of angle M
19. The distance between two schools on a map with a scale of 1:10,000, is 20 cm. What is the actual distance in kilometers between the two schools?

20. The equation of the line shown in the graph is
   A. \( x + 4y = 4 \)
   B. \( 2x - y = 4 \)
   C. \( 2x = y - 2 \)
   D. \( x - 4y + 2 = 0 \)
   E. \( 4x - y = 2 \)

21. Which of the following numbers in base two is (are) even?
   I. 110011
   II. 110010
   III. 110101
   IV. 100100
   A. I only
   B. III only
   C. I and III only
   D. II and IV only
   E. I, III and IV

22. The expression \( \frac{a}{b-c} + \frac{a}{c-b} \), where \( a \neq 0 \), is equal to
   A. 0
   B. \( \frac{2a}{b-c} \)
   C. \( \frac{a}{b^2 - c^2} \)
   D. \( \frac{a}{2b} \)
   E. 2a
23. Soda costs \( \frac{a}{b} \) cents for each bottle but there is a refund of \( \frac{b}{c} \) cents on each empty bottle. How much will Henry have to pay for \( x \) bottles if he brings back \( y \) empties?

A. \( ax + by \)  
B. \( ax - by \)  
C. \( (a - b)x \)  
D. \( (a + x) - (b + y) \)  
E. None of these

24. From a long stick of wood a man cut 6 short sticks each 2 feet long. He then found he had 1 foot left over. Which of the following would tell him the length of the original stick of wood?

A. \( 6 \times (2 + 1) \)  
B. \( (6 \times 2) + 1 \)  
C. \( (6 \div 2) - 1 \)  
D. \( (6 \times 2) - 1 \)  
E. \( (6 \div 2) + 1 \)
International Mathematics Test

Test C

Test Directions:

This short test samples many different topics in mathematics. Some problems you will probably know how to do, and some you will probably not know how to do. Do the very best you can, but do not waste time puzzling over any one question. If you get stuck on a question, skip it and go on to the next one. If you finish the test before the time is up, you can go back and work on any questions that you have skipped.

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1. Which of the following is the same as a quarter of a million?
   A. 25,250      C. 1
   B. 40,000       4,000,000
   D. 250,000      E. 2,500,000

2. \(0.40 \times 6.38\) is equal to
   A. 2.552   B. 2.452   C. 2.552   D. 24.52   E. 25.52

3. The sum of \(9\frac{4}{5}\) and \(13\frac{1}{4}\) is equal to
   A. \(22\frac{5}{9}\)   B. \(22\frac{9}{20}\)   C. 23   D. \(23\frac{1}{20}\)   E. \(23\frac{1}{5}\)

4. The ratio of 2 to 5 equals the ratio of what number to 100?

5. In a given triangle the measures of two angles in degrees are 60 and 70. What is the measure of the third angle in degrees?

6. On level ground, a boy 5 feet tall cast a shadow 3 feet long. At the same time a nearby telephone pole 45 feet high casts a shadow the length of which, in feet, is
   A. 24   B. 27   C. 30   D. 60   E. 75

7. A runner ran 3000 meters in exactly 8 minutes. What was his average speed, in meters per second?
   A. 3.75   B. 6.25   C. 16.0   D. 37.5   E. 62.5

8. On the scale above, the reading indicated by the arrow is between
   A. 51 and 52   D. 62 and 64
   B. 57 and 58   E. 64 and 66
   C. 60 and 62
9. If $x + y = 4$ and $x - y = 2$, then $x$ is equal to
   A. 0       B. 1       C. 2       D. 3       E. 6

10. One bell rings every 8 minutes, while another bell rings every 12 minutes. They have rung together once at the same moment. After how many minutes will they ring together again?
   A. for the first time?
   B. for the second time?
   C. for the tenth time?

11. At 4 o'clock, the measure of the angle between the minute hand and the hour hand of a clock, in degrees, is
   A. 30       B. 45       C. 60       D. 90       E. 120

12. Any two regular polygons with the same number of sides are
   A. congruent
   B. non congruent
   C. similar
   D. not similar
   E. equal in area

Imagine that the geometrical figures K, L, M, N and O have been drawn on a rubber sheet. The lines are assumed to have no width. The rubber sheet is stretched parallel to the X-axis while leaving all the distances measured parallel to the Y-axis unchanged. The stretching is uniform, that is, the same for every part of the sheet.

13. For which of the segments K, L, M will the length remain unchanged?
   A. only K   B. only L   C. only M   D. K and L   E. K and M

14. What will happen to the measure of angle $\theta$ of triangle N?
   A. It will remain the same.
   B. It will become larger.
   C. It will become smaller.
   D. One cannot tell from the data whether A, B, or C is correct.

15. What will happen to circle O?
   A. It will still be a circle.
   B. It will no longer be a circle.
   C. One cannot tell from the data whether A or B is correct.

16. A factory produces $m$ units per week. How many units per week will it produce after production is increased $p$ percent?
   A. $100p + m$
   B. $100m + mp$
   C. $\frac{m + mp}{100}$
   D. $m + \frac{mp}{100}$
   E. $\frac{p}{100} + m$
17. Let the symbol \( \overline{a,b} \) denote the set of integers between \( a \) and \( b \). For example, \( \overline{3,7} \) consists of the integers 4, 5, and 6. Which of the following pairs of sets has a larger number of integers in common than any of the other pairs?

A. \( \overline{0,15} \) and \( \overline{7,20} \)  
B. \( \overline{5,15} \) and \( \overline{16,30} \)  
C. \( \overline{5,14} \) and \( \overline{5,17} \)  
D. \( \overline{4,18} \) and \( \overline{8,20} \)  
E. \( \overline{0,12} \) and \( \overline{6,12} \)

18. What are all values of \( x \) for which the inequality

\[
5x + \frac{5}{3} \geq -2x - \frac{2}{3}
\]

is true?

A. \( x \geq \frac{7}{9} \)  
B. \( x \geq -\frac{1}{3} \)  
C. \( x \geq 0 \)  
D. \( x \geq \frac{7}{3} \)  
E. \( x \geq \frac{9}{3} \)

19. The symbol \( P \cap \overline{Q} \) represents the intersection of sets \( P \) and \( \overline{Q} \) and the symbol \( P \cup \overline{Q} \) represents the union of sets \( P \) and \( \overline{Q} \). Which of the following represents the shaded portion of the diagram below?

A. \( (X \cap Y) \cup Z \)  
B. \( X \cup (Y \cap Z) \)  
C. \( X \cap (Y \cup Z) \)  
D. \( (X \cap Y) \cap Z \)  
E. \( (X \cup Y) \cap Z \)
20. If, in the figure below, PQ and RS are intersecting straight lines, then \(x + y\) is equal to

\[
\begin{array}{c}
\text{P} \\
\text{Q} \\
\text{R} \\
\text{S}
\end{array}
\]

\(150^\circ\)

\(x + y\)

A. 15  
B. 30  
C. 60  
D. 180  
E. 300

21. Each of 9 boys had \(t\) marbles. In order to play a game, they divided the marbles among 12 boys in such a way that each had the same number. How many marbles did each of the 12 have?

A. \(\frac{3t}{4}\)  
B. \(t - 3\)  
C. \(\frac{4t}{3}\)  
D. \(9t - 12\)  
E. \(12t - 9\)

22. The length of the circumference of the circle on the right with center at \(O\) is 24 and the length of arc RS is 4. What is the measure in degrees of the central angle ROS?

A. 24  
B. 30  
C. 45  
D. 60  
E. 90

23. Given any fraction whose numerator is less than the denominator, if you then add 2 to both the numerator and the denominator, the new fraction is

A. equal to the original fraction  
B. larger than the original fraction  
C. twice the original fraction  
D. smaller than the original fraction  
E. 1 more than the original fraction
INTERNATIONAL MATHEMATICS TEST

PRACTICE QUESTIONS

The following six examples will give you practice in the method of answering questions on the test. (Use the scratch paper if you need to do any figuring.)

I.

Questions will be answered in two ways. Several answers will be given for some questions, and you are to choose the right one, like this:

1. What is the sum of 2 and 2?
   A. 2  B. 3  C. 4  D. 5  E. 6

   The correct answer is 4. 4 is choice C. On your answer sheet, at the bottom of the page where you have written your name, find number 1. In that row, darken the space between the dotted lines to the right of C.

   Do the following practice examples and mark the correct answer for each question on your answer sheet.

2. What does 9 minus 4 equal?
   A. 2  B. 3  C. 5  D. 6  E. 7

3. 7 x 2 = ?
   A. 5  B. 9  C. 12  D. 13  E. 14

4. Add:  
   \[ \begin{array}{c}
   21 \\
   +7 \\
   \hline
   32 \\
   \end{array} \]
   A. 40  B. 50  C. 59  D. 60  E. 70

II.

For other questions no choices will be given. You are to write your answer on the line provided on the answer sheet, like this:

5. The product of 3 and 4 is ________.

   The right answer is 12. Write the number 12 on the line to the right of number 5 on the answer sheet.

   Now solve number 6, and write your answer on the answer sheet.

International Mathematics Test

Test 4

Test Directions:

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1. In \( \triangle PQR \) at right, which of the following conditions is sufficient to prove that the altitude from \( P \) coincides with the bisector of angle \( \angle P \)?
   
   A. \( x = y \)  \hspace{1cm} D. \( PR = RQ \)
   B. \( y = z \)  \hspace{1cm} E. \( PQ = QR \)
   C. \( x = z \)

2. Which of the following is equivalent to \( x(x + a) - a(x - a) \)?
   
   A. \( (x + a)(x - a)^2 \)  \hspace{1cm} D. \( (x + a)^2 \)
   B. \( (x + a)^2(x - a) \)  \hspace{1cm} E. \( x^2 + a^2 \)
   C. \( (x + a)^3 \)

3. Which of the following is equal to \( a^{-3} \)?
   
   A. \( (-1)a^3 \)  \hspace{1cm} C. \( \frac{1}{a^3} \)
   B. \( -\frac{1}{a^3} \)  \hspace{1cm} D. \( -3a \)
   E. \( a - 3 \)

4. Suppose that the symbol \( * \) denotes an operation on the integers, and suppose that for all integers \( x \) and \( y \), \( x * y = y * x \). It follows from this that, for integers \( a, b, \) and \( c \), where \( + \) denotes the usual operation of addition,
   
   A. \( (a + b) * (a + c) = (c + b) * (c + a) \)
   B. \( a * (b + c) = (a * b) + c \)
   C. \( (a + b) * (a + c) = (a + c) * (a + b) \)
   D. \( (a + b) * c = (a + c) * b \)
   E. If \( a * b = c \), then \( a * c = b \)

5. If the chord of a circle equals the radius, its minor arc equals how many degrees?
   
   A. 15 \hspace{1cm} B. 30 \hspace{1cm} C. 45 \hspace{1cm} D. 60
   
   E. It cannot be determined from the information given.
6. In the figure at right, \( \triangle ABC \) is a triangle and \( BD = CE \). Triangles \( BCD \) and \( CBE \) are
A. congruent by SSS
B. congruent by SAS
C. congruent by ASA
D. similar by SAS
E. not necessarily congruent or similar

7. The expression \( \left( \frac{5}{4} \right) \left( \frac{2}{3} \right) - \left( \frac{1}{2} \right) \left( \frac{2}{3} \right) \) is equal to
A. \( \frac{1}{2} \)
B. \( -\frac{1}{12} \)
C. \( -\frac{1}{2} \)
D. \( -\frac{7}{6} \)
E. None of these

8. Given: line \( PQ \) intersects line \( RS \) at \( O \)
Prove: \( x = z \)

Statement
1. \( x + y = 180 \)
2. \( z + y = 180 \)
3. \( x + y = z + y \)
4. \( x = z \)

In the proof above, what is the authority or reason for passing from statements 1 and 2 to statement 3?
A. If two angles are equal, their supplements are equal.
B. If for three numbers \( a = b \) and \( b = c \), then \( a = c \).
C. If equals are added to equals, the results are equal.
D. If equals are subtracted from equals, the results are equal.
E. If two lines intersect, the measures of the opposite angles are equal.

9. In the circle below, chord \( AB \) is 12 inches long and 8 inches from the center \( O \). What is the length, in inches, of the radius of the circle?

A. \( \sqrt{80} \)
B. 10
C. \( \sqrt{208} \)
D. 16
E. 20
10. Which of the following illustrates the distributive property?
   A. $2 + 6 = 6 + 2$  
   B. $(4 + 2) + 6 = 4 + (2 + 6)$  
   C. $(4 \times 2) + 6 = (2 \times 4) + 6$  
   D. $4 \times (2 \times 6) = (4 \times 2) \times 6$  
   E. $4 \times (2 + 6) = (4 \times 2) + (4 \times 6)$

11. What value of $k$ makes the following equations consistent?
   \[
   \begin{align*}
   x &= \frac{2}{3}k \\
   y - 6x &= 6k \\
   y &= 6 - 2k
   \end{align*}
   \]
   A. $-\frac{3}{4}$  
   B. $\frac{1}{2}$  
   C. $\frac{3}{4}$  
   D. 1  
   E. 3

12. For what real numbers $x$ is $x^2 - 2x - 3 > 0$?
   A. $x < -1$  
   B. $x > 3$  
   C. $-1 < x < 3$  
   D. $-3 < x < 1$  
   E. $x < -1$ or $x > 3$

13. What is the least common multiple of the polynomials $s^2 + s - 12$ and $s^2 - 5s + 6$?
   A. $(s - 2)(s - 3)(s + 4)$  
   B. $(s - 2)(s - 3)^2(s + 4)$  
   C. $(s - 2)(s + 3)(s - 4)$  
   D. $(s - 2)(s - 3)(s + 3)(s - 4)$  
   E. $(s + 2)(s - 3)(s + 3)(s - 4)$

14. What is (are) the solution(s), in real numbers, of the equation $x + 1 = \sqrt{x + 7}$?

15. A motor boat can go 8 miles down-stream in 40 minutes. It takes one hour to travel the same distance up-stream. Find (a) the speed of the boat, and (b) the rate of the stream in miles per hour.
16. If \( \log x = \log 1 + \log 2 + \log 3 + \log 4 + \log 5 \), then \( x \) is
   A. 6      B. 15      C. 36      D. 55      E. 120

17. For what value of \( k \) will the roots of the equation
   \[ x^2 + 6x + k = 0 \]
   be equal?

18. To construct the bisector of an angle \( \angle POQ \), the first step is to mark off with compasses the intersections of a circle with center at \( O \) and the two rays \( OP \) and \( OQ \), as shown below.

   ![Diagram of angle bisector]

   For the simplest construction with ruler and compass the next step is to
   A. Draw the line \( RS \)
   B. Find the intersection of circles with equal radii and centers \( R \) and \( S \)
   C. Erect perpendiculars to \( OP \) and \( OQ \) at \( R \) and \( S \)
   D. Draw the median to the side \( RS \) of the triangle \( ORS \)
   E. Drop a perpendicular from \( R \) to \( OQ \)
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1. If \( a = 20, \ b = 0, \ c = 10, \ x = 8, \ y = 12, \) then the value of 
\[ 2aby + 2cx \] is

A. 100  
B. 160  
C. 400  
D. 640  
E. none of

these

2-5. For each of the following equations or pairs of equations,
concerned with real numbers, mark on the answer sheet

A. if there is no solution
B. if there is one solution
C. if there are two solutions
D. if there are three solutions
E. if there are more than three solutions

2. \( x + y = 12, \ x - y = 4 \)

3. \( m + n = 2, \ 3m + 3n = 9 \)

4. \( x^2 - 5x + 6 = 0 \)

5. \( 3p + q = 16 \)

6. If \( xy = 1 \) and \( x \) is greater than 0, which of the following
statements is true?

A. When \( x \) is greater than 1, \( y \) is negative.
B. When \( x \) is greater than 1, \( y \) is greater than 1.
C. When \( x \) is less than one, \( y \) is less than 1.
D. As \( x \) increases, \( y \) increases.
E. As \( x \) increases, \( y \) decreases.
7. In the figure on the right, 

\[ KX = \frac{1}{3} KL \quad \text{and} \quad KY = \frac{1}{3} KM. \]

Which of the following statements are true?

I. \( XY = \frac{1}{3} LM \)

II. Line \( XY \) is parallel to line \( LM \)

III. Area \( KXY = \frac{1}{3} \text{ area } KLM \)

IV. Area \( KXY = \frac{1}{9} \text{ area } KLM \)

A. I and II only
B. II and III only
C. I and III only
D. I, II and III only
E. I, II, and IV only

8. In the figure on the right, \( m \) represents a plane, and \( PQ \) is a line segment which is perpendicular to the plane at the point \( Q \). Points \( A, B \) and \( C \) lie on the plane. If \( QA = QB = QC \), then the triangles \( PQA \), \( PQB \), and \( PQC \) are

A. congruent by SAS
B. congruent by SSA
C. congruent by ASA
D. similar but not congruent
E. neither similar nor congruent

9. In the figure below, \( PQ \perp OQ \), and \( RS \perp OQ \). If the measure of \( OQ \) and of \( OR \) equal 1 and \( \theta \) is the measure of \( \angle POQ \), then the measure of segment \( PQ \) is equal to

A. \( \sin \theta \)
B. \( \cos \theta \)
C. \( \tan \theta \)
D. \( 2 \sin \theta \)
E. \( 1 - \cos \theta \)
Questions 10 and 11 are based upon the graph of a quadratic equation which is shown in the figure above.

10. For what value of $x$ is the quadratic function a minimum?
   A. $-1$  B. $-1/2$  C. $1/2$  D. 1  E. $1 \frac{1}{2}$

11. The range of values of $x$ for which the function represented by the straight line $MN$ exceeds the quadratic function is
   A. $-1 < x < 1$  D. $x > 0$
   B. $x < -1$ and $x > 1$
   C. $-3/4 < x < 1\frac{1}{4}$  E. $x > y$

12. A square plate of the largest possible size is cut from a circular plate of 16 cm. diameter. The area of the square plate (in sq. cm.) will be
   A. 64  B. 96  C. 128  D. 192  E. 256

13. The locus of all mid points of chords drawn from a point of a circle is
   A. a semi-circle  D. an oblong
   B. a circle  E. none of these
   C. a straight line
14. A piece of wire 52 inches long is cut into two parts and each part is bent to form a square. The total area of the two squares is 97 square inches. What is the length in inches of the side of the smaller square?

15. The complex number \((1 + i)^2\) is equal to
   A. 0  B. 2  C. 2i  D. 1 + 1  E. 2 + 2i

16. Given \(\log_b 2 = \frac{1}{3}\), \(\log_b 32\) is equal to
   A. 2  B. 5  C. \(-\frac{3}{5}\)  D. \(\frac{5}{3}\)  E. \(\frac{3}{\log_3 32}\)

17. Below there are several definitions of new operations named \(*\) in terms of the usual operations on real numbers. For which of the definitions is the property \(y * x = x * y\) valid for all positive real numbers \(x\) and \(y\)?
   A. \(x * y = \frac{x}{y}\)  D. \(x * y = \frac{xy}{x + y}\)
   B. \(x * y = x - y\)  E. \(x * y = x^2 + xy + y^2\)
   C. \(x * y = x(x + y)\)

18. Solve the equation
   \[\sqrt{x + 5} - \sqrt{x - 3} = \sqrt{x}\]

19. The graph on the right is the representation of one of the following equations. Which one does it represent?
   A. \(y = (1 - x)(x - 2)\)
   B. \(y = (1 - x)(2 - x)\)
   C. \(y = (1 - x)(2 - x)^2\)
   D. \(y = (1 - x)^2(x - 2)\)
   E. \(y = (1 - x)^2(2 - x)\)
20. The expression
\[
\frac{2}{\sqrt{5}} + \frac{\sqrt{45}}{5} + \frac{1}{\sqrt{5} - 2}
\]
is equal to
A. \(2\sqrt{5} + 2\)  
B. \(2\sqrt{5} - 2\)  
C. 2  
D. \(2\sqrt{5}\)  
E. \(2 - 2\sqrt{5}\)

21. Chords of the same length are drawn in two circles of unequal radii. Which of the following is true?
A. The chord in the larger circle could be equal to the radius of the smaller circle.
B. The chord in the smaller circle could not be a diameter.
C. The distance from the center to the chord is less in the larger circle.
D. The minor arc intercepted on the larger circle is longer.
E. The minor arc intercepted on the larger circle subtends the greater angle at the center.
International Mathematics Test

Test 6

Test Directions:

This short test samples many different topics in mathematics. Some problems you will probably know how to do, and some you will probably not know how to do. Do the very best you can, but do not waste time puzzling over any one question. If you get stuck on a question, skip it and go on to the next one. If you finish the test before the time is up, you can go back and work on any questions that you have skipped.

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1. On the outside of two sides of a rectangular plot 25 by 20 meters there is a path of uniform width. The area covered by the path equals one half the area of the plot. What is the width of the path in meters?

2. According to one plan for travelling to Mars, the round trip would take almost exactly three years, including a stay on Mars of 449 earth days. If one must go 34,000,000 miles each way on the trip, which of the following can be used to give an estimate of the average speed of travel in miles per hour?

   A. \( \frac{(3 \times 365 - 449) \times 24}{34,000,000} \)  
   B. \( \frac{(3 \times 365 - 449) \times 24}{34,000,000 \times 2} \)  
   C. \( \frac{34,000,000}{(3 \times 365 - 449) \times 24} \)  
   D. \( \frac{34,000,000 \times 24}{2 \times (3 \times 365 - 449)} \)  
   E. \( \frac{2 \times 34,000,000}{(3 \times 365 - 449) \times 24} \)

3. Of three wires, each 36 in. long, one is bent into a square, another into a rectangle with length and width in the ratio of 2:1 and the third into an equilateral triangle. Which one of the following statements describes the correct relationship between the enclosed areas?

   A. The area of the square is the greatest and that of the triangle is the least.
   B. The area of the rectangle is the greatest and that of the triangle is the least.
   C. The area of the triangle is the greatest and that of the square is the least.
   D. The area of the triangle is the greatest and that of the rectangle is the least.
   E. The areas of the square and the rectangle are the same, but the area of the triangle is less than that of the square or the rectangle.

4. If \( 3^x + y = 81 \) and \( 25^2 = 5 \), then \( y \) is

   A. 0  
   B. 2  
   C. 3  
   D. \( \frac{7}{2} \)  
   E. \( \frac{15}{4} \)
5. A certain number of students are to be accommodated in a hostel. If 2 students share each room, then 2 students will be left without any room. If 3 students share each room, then 2 rooms will be left unoccupied. How many rooms are there in the hostel?

6. Four persons whose names begin with different letters are placed in a row, side by side. What is the probability that they will be placed in alphabetical order from left to right?
   A. 1/120  B. 1/24  C. 1/12  D. 1/6  E. 1/4

7. A number is the multiplicative inverse of another number if the product of the two numbers is 1. Which of the following sets of numbers is identical to the set of its multiplicative inverses?
   A. {1, 2, 3}  D. {2, 3, 5, 1/2, 1/3}
   B. {1, 1/3}  E. {2, 3, 2/3}
   C. {1, 2, 1/3}

8. In the figure above, the circle with center C is internally tangent at point T to the circle with center O. P is a point on the larger circle such that TP is not a diameter. If TP intersects the smaller circle at A, then what additional information is needed to prove that AC and PO are parallel?
   A. None  D. CO = 2TC
   B. PO = 2AC  E. TA = AC
   C. TA = AP
9. Which of the following is (are) true for all values of \( \theta \) for which the functions are defined?

I. \( \sin(-\theta) = -\sin \theta \)
II. \( \cos(-\theta) = \cos \theta \)
III. \( \tan(-\theta) = -\tan \theta \)

A. I only  
B. II only  
C. III only  
D. I and III only  
E. II and III only

10. A radio-active element decomposes according to the formula
\[ y = y_0 \cdot e^{-k \cdot t} \]
where \( y \) is the mass of the element remaining after \( t \) days and \( y_0 \) is the value of \( y \) for \( t = 0 \). Find the value of the constant \( k \) for an element whose half-life (i.e., time to decompose half of the material) is 4 days.

A. \( \frac{1}{4} \log_e 2 \)  
B. \( \log_e \frac{1}{2} \)  
C. \( \log_2 e \)  
D. \( (\log_e 2)^{1/4} \)  
E. \( 2e^4 \)

11. A stationer wants to make a card 8 cm. long and of such a width that when it is cut into halves, the original width becomes the length and the shape of each half is similar to the original card. What width, in cm., should he make the original card?

A. 4  
B. \( 4\sqrt{2} \)  
C. \( 5\sqrt{2} \)  
D. \( 5\sqrt{3} \)  
E. 6

12. The arithmetic mean or average of one group of 100 pupils is exactly 80 and the mean of another group of 50 pupils is exactly 65. What is the mean of the combined group of 150 pupils?

A. 79  
B. 72.5  
C. 75  
D. 77.5  
E. It is impossible to determine exactly.
13. In the diagram on the right, the numbers represent regions.

The circle X represents the set of regular polygons
The circle Y represents the set of quadrilaterals
The circle Z represents the set of equilateral triangles
Which are the parts of the schema that are empty [have no elements]? 
A. 1, 3 and 5  D. 1, 3 and 7  
B. 2, 3 and 4  
C. 1, 6 and 7  E. 3, 6 and 7

14. In the figure on the right, FGHJ is a parallelogram. Which of the following statements is a condition which implies that FGHJ is a rectangle? 
A. JF = GH  
B. \( \angle HJG = \angle JGF \)  
C. \( \angle HJF = \angle JHG \)  
D. \( \angle HJF \) and \( \angle JGH \) are supplementary.  
E. HF and JG are perpendicular bisectors of each other.

15. What is the sum of the infinite geometric series 
\[ 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots ? \]
A. \( \frac{5}{8} \)  B. \( \frac{2}{3} \)  C. \( \frac{3}{5} \)  D. \( \frac{3}{2} \)  E. \( \infty \)

16. A freight train travelling at 50 miles per hour leaves a station 3 hours before an express train which travels in the same direction at 90 miles an hour. How many hours will it take the express train to overtake the freight train?
A. \( \frac{5}{9} \)  B. \( \frac{9}{5} \)  C. \( \frac{12}{5} \)  D. \( \frac{15}{4} \)  E. \( \frac{18}{4} \)
17. In right triangle $PQR$ (at the right) the measure of $PQ$ is $4$ and $\theta$ can be any angle between $30^\circ$ and $45^\circ$. What are all possible values for $x$, the length of $RQ$?

A. $0 < x < 4$
B. $\frac{1}{2} < x < \frac{\sqrt{2}}{2}$
C. $\frac{1}{2} < x < \frac{\sqrt{3}}{2}$
D. $2 < x < 2\sqrt{2}$
E. $2 < x < 2\sqrt{3}$
International Mathematics Test

Test 7

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1. The expression \(|x - 1| = 1\) implies that
   
   A. \(x\) is between 0 and 2  
   B. \(x\) is either 0 or 2  
   C. \(x\) is less than 2  
   D. \(x\) is 0  
   E. \(x\) is 2

2. When \((1 + p)^5\) is expanded, the coefficient of \(p^4\) is
   
   A. 6  
   B. 10  
   C. 15  
   D. 20  
   E. 30

3. What is the converse of the statement, "If two angles are vertical, then they are equal"?
   
   A. If two angles are vertical, then they are not equal.  
   B. If two angles are equal, then they are vertical.  
   C. If \(\angle x\) and angle \(\angle y\) are vertical angles, then \(\angle x = \angle y\).  
   D. If two angles are not vertical, then they are not equal.  
   E. If two angles are not equal, then they are not vertical.

4. Suppose you have proved the two theorems:
   
   I. If \(p\) then \(q\).  
   II. If \(s\) then not \(q\).
   
   Which of the following theorems is implied by theorems I and II?
   
   A. If \(p\) then \(s\).  
   B. If not \(p\) then not \(q\).  
   C. If \(p\) or \(q\) then \(s\).  
   D. If \(s\) then not \(p\).  
   E. If not \(s\) then \(q\).

5. A train travelled a certain distance at a constant speed. Had the speed been 8 m.p.h. greater, the trip would have taken one hour less. Had the speed been 12 m.p.h. less the trip would have taken two hours more. How many miles did the train go?
6. A wholesale merchant bought a television set at a certain price and then sold it to a retail merchant at an increase of \( P \) percent of this price. The retail merchant sold the set to a consumer for \( P \) percent more than he paid for it. If the customer paid 65 percent more than the price originally paid by the wholesale merchant, then \( P \) satisfies the equation:

A. \( 1 + \frac{2P}{100} = 1.65 \)  
B. \( \left( 1 + \frac{P}{100} \right)^2 = 1.65 \)  
C. \( 1 + \left( \frac{P}{100} \right)^2 = 1.65 \)  
D. \( 1 + P^2 = 1.65 \)  
E. \( 1 + 2P = 1.65 \)

7. If a relation \( R \) is such that \( xRy \) and \( yRz \) implies \( xRz \) for each \( x, y, \) and \( z \) of a given set, the relation \( R \) is said to be transitive on that set. Which of the following relations are transitive?

I. "is father of"
II. "is contemporary of"
III. "is admirer of"
IV. "is multiple of"
V. "is perpendicular to"

A. II, IV and V  
B. I and II  
C. II, III and IV  
D. II and IV  
E. V only

8. In the figure shown to the right, which vector is a graphical representation of the complex number \( 4 - 2i \)?

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The answers are:  
6. A  
7. C  
8. A
9. Solve $0 < x^2 - 3x + 3 < 7$

10. A relation $R$ from a set $S$ to a set $T$ is a function if and only if given an $x \in S$ there exists at most one $y \in T$ such that $xRy$.

Which of the following relations are functions?

I. $x$ divides $y$
II. $x$ has $y$ for mother
III. $x$ is parallel to $y$
IV. $x$ has $y$ for double
V. $x$ has $y$ as majorant (i.e., $x < y$)
VI. $x^2 = y$

A. I, II, and III
B. II, IV, and V
C. II, IV, and VI
D. IV, V, and VI
E. I, IV, and V

11. What is the equation whose roots are the squares of the roots of $x^2 - 5x + 3 = 0$?

A. $x^2 - 19x + 9 = 0$
B. $x^2 + 19x + 9 = 0$
C. $x^2 - 20x + 9 = 0$
D. $x^2 + 19x - 9 = 0$
E. $x^2 - 9x + 19 = 0$
Questions 12 and 13. Six operations are defined as follows:

\[
A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \\
D = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \quad E = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \quad F = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}
\]

The operation \( A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \), for example, means that the numbers in the upper row are transformed into the digits in the lower row, so that \( 1 \rightarrow 2 \) (1 becomes 2), \( 2 \rightarrow 3 \) (2 becomes 3), and \( 3 \rightarrow 1 \) (3 becomes 1). \( A \cdot B \) shows that operation \( B \) is to be performed after operation \( A \); that is, according to \( A \), \( 1 \rightarrow 2 \), \( 2 \rightarrow 3 \), \( 3 \rightarrow 1 \), and then, according to \( B \), \( 2 \rightarrow 1 \), \( 3 \rightarrow 2 \), \( 1 \rightarrow 3 \). Therefore \( A \cdot B \) will be \( 1 \rightarrow 2 \rightarrow 1 \), \( 2 \rightarrow 3 \rightarrow 2 \), and \( 3 \rightarrow 1 \rightarrow 3 \). This produces the same outcome as

\[
F = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}; \text{ let us write this } A \cdot B = F.
\]

In like manner, \( A \cdot C \) is \( 1 \rightarrow 2 \rightarrow 3 \), \( 2 \rightarrow 3 \rightarrow 2 \), \( 3 \rightarrow 1 \rightarrow 1 \), and is the same as \( D \); that is to say, \( A \cdot C = D \).

12. Which one operation is equal to \( C \cdot D \)?

13. What operation must be performed after operation \( B \) so that the combined operations are to be the same as operation \( F \)?

14. If \( x \) and \( y \) belong to the set of real numbers and sets \( P \), \( Q \) and \( R \) are defined as follows,

\[
P = \{(x,y) \mid x^2 + y^2 = 4\} \\
Q = \{(x,y) \mid x - y = 2\} \\
R = \{(x,y) \mid (x^2 + y^2 - 4)(x - y - 2) = 0\},
\]

which of the following is true?

A. \( R = P \cap Q \)  
B. \( R = P \cup Q \)  
C. \( R = \{(2,0)(0,2)(-2,0)(0,-2)\} \)  
D. \( R = \{\} \) (the empty set)  
E. \( R = \{(2,0)(0,-2)\} \)
15. The value \[
\begin{vmatrix}
4 & 2 & 1 \\
0 & 0 & 1 \\
1 & 1 & 0 \\
\end{vmatrix}
\]
is

A. -2  B. 0  C. 2  D. 7  E. 9

16. Each root of \(x^2 - 2x + 5 = 0\) differs from the cube of the other by a positive constant \(c\). What is the value of \(c\)?

17. Two of the roots of the equation \(x^4 - 27x^2 - 14x + 120 = 0\) are 2 and 5. Find the two other roots of the equation.
International Mathematics Test

Test 8

Test Directions:

This short test samples many different topics in mathematics. Some problems you will probably know how to do, and some you will probably not know how to do. Do the very best you can, but do not waste time puzzling over any one question. If you get stuck on a question, skip it and go on to the next one. If you finish the test before the time is up, you can go back and work on any questions that you have skipped.

Do not make any marks on the test booklet. Use the separate scratch paper for your figuring. Be sure to mark the correct row on your answer sheet for each question.
1. If \( \log_a 8 = \frac{3}{2} \), what is the value of \( a \)?
   A. 2/3  B. 2  C. 4  D. 5  E. 6

2. If \( x \) and \( y \) are real numbers, for which \( x \) can you define \( y \) by
   \[ y = \frac{x}{\sqrt{9 - x^2}} \]
   A. All \( x \) except \( x = 3 \)  D. \(-3 < x < 3\)
   B. All \( x \) except \( x = 3 \) and \( x = -3 \)  E. \( x < 3 \)
   C. \( x < -3 \) and \( x > 3 \)

3. A set of 24 cards is numbered with the positive integers from 1 to 24. If the cards are shuffled and if only one is selected at random, what is the probability that the number on the card is divisible by 4 or 6?
   A. 1/6  B. 5/24  C. 1/4  D. 1/3  E. 5/12

4. An angle \( \theta \) is known to be between \( 180^\circ \) and \( 270^\circ \) and
   \[ \cos^2 \theta = \frac{16}{25} \]. The value of \( \sin 2\theta \) is then
   A. \(-\frac{24}{25}\)  B. \(-\frac{15}{25}\)  C. \(-\frac{7}{25}\)  D. \(\frac{7}{25}\)  E. \(\frac{24}{25}\)

5. For some functions the relationship holds that
   \( f(x + y) = f(x) + f(y) \) for all numbers \( x \) and \( y \). For example, when \( f(x) = 2x \), then \( f(x + y) = 2(x + y) = 2x + 2y = f(x) + f(y) \), i.e., \( f(x + y) = f(x) + f(y) \). We call such functions as these additive. Which of the following functions is additive by this definition?
   A. \( f(x) = x^2 \)  D. \( f(x) = 2^x \)
   B. \( f(x) = \sin x \)  E. None of them are additive
   C. \( f(x) = \log_{10} x \)
6. If determinants are used to solve the system of equations

\[
\begin{align*}
2x + y &= 3 \\
x + 4y &= 7,
\end{align*}
\]
then \( y \) is equal to

A. \[
\begin{vmatrix}
2 & 1 \\
1 & 4
\end{vmatrix}
\]
B. \[
\begin{vmatrix}
2 & 1 \\
1 & 4
\end{vmatrix}
\]
C. \[
\begin{vmatrix}
2 & 3 \\
1 & 7
\end{vmatrix}
\]
D. \[
\begin{vmatrix}
3 & 1 \\
4 & 1
\end{vmatrix}
\]
E. \[
\begin{vmatrix}
3 & 7 \\
2 & 1
\end{vmatrix}
\]

---

Items 7-9 refer to the information below.

Consider the following abstract mathematical system:

**Undefined terms**: elements \( a, b, c \ldots \) of class \( C \);
operations \( \lambda \) and \( * \); relation \( = \), having the conventional meaning of "equals".

**Postulates**: If \( a, b, \) and \( c \) are any elements of \( C \), then

1. \( a\lambda b \) and \( a*b \) are elements of \( C \).
2. \( a\lambda b = b\lambda a \).
3. \( a*(b*c) = (a*b)*c \).
4. \( a*b \neq b*a \), provided \( a \neq b \).
5. \( a\lambda (b*c) = (a\lambda b)*(a\lambda c) \).

**DIRECTIONS**: Answer each item, using the code

A - if the proposition follows logically from the postulates.
B - if the proposition is inconsistent with the postulates (i.e., contradicts the postulates).
C - neither A nor B (i.e., the proposition neither follows from the postulates nor is contradicted by them).

7. \( (a*b)\lambda c = (c\lambda a)*(c\lambda b) \)
8. \( (a*b)\lambda c = (a\lambda c)*(b\lambda c) \)
9. \( a\lambda (b*c) = (a\lambda c)*(a\lambda b) \)
10. The graph of \( y = f(x) \) is a parabola with axis parallel to the Y-axis. If the maximum value of \( y \) is 2, and if the parabola crosses the X-axis at \( x = -\frac{1}{2} \) and at \( x = \frac{3}{2} \), then its equation is

A. \( y = -2x^2 + 2x + \frac{3}{2} \)  
B. \( y = -4x^2 - 4x + 3 \)  
C. \( y = -4x^2 + 4x + 3 \)  
D. \( y = 4x^2 - 4x - 3 \)  
E. \( y = 4x^2 + 4x - 3 \)

11. For what values of the real number \( x \) is \( y = \frac{1}{x} \) a decreasing function?

A. No \( x \)  
B. \( x < 0 \)  
C. \( x \neq 0 \)  
D. \( x > 0 \)  
E. All \( x \)

12. Solve: \( 2 \cdot 7^{2+x} + 3 \cdot 7^{3+x} = 161 \)

13. Given two arbitrary sets \( X \) and \( Y \). Which of the following sets is equivalent to the set \( (X \cup Y) \cap (X \cap Y) \)?

A. \( X \)  
B. \( Y \)  
C. \( X \cup Y \)  
D. \( X \cap Y \)  
E. \( (X \cup Y) \cup (X \cap Y) \)

14. Consider the matrices

\[
A = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 0 \\ y & 1 \end{pmatrix}
\]

where \( x \) and \( y \) are real numbers and \( x^2 + y^2 \neq 0 \).

For which values of \( x \) and of \( y \) is the product of the matrices commutative?

I. \( x = 0 \)
II. \( y = 0 \)
III. \( x = y \)

A. Only I  
B. Only II  
C. Only III  
D. Both I and II  
E. I, II, and III
15. Calculate $\arcsin \frac{1}{2} + \arcsin \frac{1}{\sqrt{2}}$ [Arc sin means "angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sin is"]

A. $\frac{5\pi}{12}$  
B. $\frac{7\pi}{18}$  
C. $\frac{\pi}{3}$  
D. $\arcsin \left( \frac{1 + \sqrt{2}}{2} \right)$  
E. $\arcsin \frac{\sqrt{3}}{2}$

16. For what values of $x$ is the function

$$\frac{(1-x)(1+3x)}{(2x-1)(x-2)}$$

positive?
International Mathematics Test

Test 9

Test Directions:

This short test samples many different topics in mathematics. Some problems you will probably know how to do, and some you will probably not know how to do. Do the very best you can, but do not waste time puzzling over any one question. If you get stuck on a question, skip it and go on to the next one. If you finish the test before the time is up, you can go back and work on any questions that you have skipped.

Do not make any marks on the test booklet. Use the separate scratch paper for your figuring. Be sure to mark the correct row on your answer sheet for each question.

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Hamburg, Germany

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1. In a Cartesian coordinate system, what is the equation of the straight line passing through the point (0, -5) and parallel to the straight line whose equation is $y = 2x + 3$?
   A. $x + 2y + 5 = 0$
   B. $2x - y - 5 = 0$
   C. $2x + 3 = -5$
   D. $2x - 5y + 3 = 0$
   E. $2x + y + 5 = 0$

2. An open cylindrical vessel of capacity $9000\pi$ c.c. is to be made with the curved surface of sheet metal and a wooden base. If the weight of 1 sq. cm. of the metal is three times the weight of 1 sq. cm. of the wood, each being of uniform small thickness, what will be the radius of the vessel (in cms.) when its total weight is a minimum?

3. The derivative with respect to $x$ of $\frac{4}{\sqrt{3x - 4}}$ is
   A. $12\sqrt{3x - 4}$
   B. $4\sqrt{3}$
   C. $\frac{-2}{(3x - 4)^{3/2}}$
   D. $\frac{-6}{(3x - 4)^{3/2}}$
   E. $6\sqrt{3x - 4}$

4. The value of $\int_{0}^{1} \frac{dx}{x^2 - 5x + 6}$ is:
   A. $\frac{1}{2} \log_e 2$
   B. $\frac{1}{3}$
   C. $\log_e \frac{4}{3}$
   D. $\tan \frac{-11}{4}$
   E. $\frac{1}{2}$

5. $\int (x - 1)^3 dx$ is equal to
   A. $2(x - 1) + c$
   B. $\frac{1}{2} (x - 1)^2 + c$
   C. $\frac{1}{3} (x - 1)^3 + c$
   D. $\frac{1}{3} (x^3 - x) + c$
   E. $\frac{(x - 1)^3}{x} + c$
6. Determine $k$ so that the graph of the function $y = 3x^3 + 6x^2 + kx + 9$ has a point of inflection and a horizontal tangent for the same value of $x$.

7. What is the equation in $x$ and $y$ of the curve with parametric equations $x = t + \frac{1}{t}$, $y = t - \frac{1}{t}$?

   A. $x + y = 1$
   B. $x + y = 2$
   C. $x^2 + y^2 = 4$
   D. $x^2 - y^2 = 4$
   E. $2x^2 - y^2 = 4$

8. The graph of a polynomial function of $x$ is shown in the diagram above, the equation of the curve being $y = f(x)$. Which of the following statements is (are) true for that part of the curve for which $a \leq x \leq b$?

   I. $f''(c) = 0$ for some value $c$ between $a$ and $b$.
   II. $\frac{f(b) - f(a)}{b - a} = f'(c)$ for some value $c$ between $a$ and $b$.
   III. If there is a point of inflexion at $Q$, $f''(b)$ can have no value but 0.
   IV. $\int_a^b f(x) \, dx < \frac{1}{2} (b - a) [f(a) + f(b)]$

   A. All four
   B. II, III and IV
   C. I and II
   D. I and III
   E. II and IV
9. Given that \( \frac{dy}{dx} = x^2 - 5 \), and \( y = 1 \) when \( x = 2 \).

What is the value of \( y \) when \( x = 0 \)?

A. \(-5/3\)  
B. \(-2/3\)  
C. \(1/3\)  
D. \(25/9\)  
E. \(31/9\)

Problems 10 and 11 are based on the figure shown below, which shows a graph of \( y = f(x) \), \( a \) being less than \( b \). \( S_1 \) is the area enclosed by the \( x \)-axis, \( x = a \), and \( y = f(x) \). \( S_2 \) is the area enclosed by the \( x \)-axis, \( x = b \), and \( y = f(x) \). \( S_1 \) and \( S_2 \) are to be considered positive.

10. The value of \( \int_a^b f(x) \, dx \) is

A. \( S_1 + S_2 \)  
B. \( S_1 - S_2 \)  
C. \( S_2 - S_1 \)  
D. \( |S_1 - S_2| \)  
E. \( \frac{1}{2}(S_1 + S_2) \)

11. The value of \( \int_a^b |f(x)| \, dx \) is

A. \( S_1 + S_2 \)  
B. \( S_1 - S_2 \)  
C. \( |S_2 - S_1| \)  
D. \( |S_2| - |S_1| \)  
E. \( \frac{1}{2}(S_1 + S_2) \)

12. The function \( f(x) = \frac{x^2 - 1}{x - 1} \) is defined and continuous for all \( x \) except \( x = 1 \). What value must be assigned to \( f(x) \) for \( x = 1 \) in order that the function be continuous there?
13. Find the difference $\overrightarrow{b} - \overrightarrow{a}$ of the vectors $\overrightarrow{a} = (4,2)$ and $\overrightarrow{b} = (0,3)$.

A. (-4, -2)  
B. (-4, 1)  
C. (4, -1)  
D. (4, 2)  
E. (4, 5)

14. In a triangle with area $\overrightarrow{a}$, the midpoints of the three sides are joined so as to form a new triangle. In the triangle thus constructed, another new triangle is inscribed in the same way. This process is continued indefinitely. What is the sum of all of the areas of this sequence of triangles, including the original one?

A. $\frac{9a}{7}$  
B. $\frac{4a}{3}$  
C. $\frac{7a}{5}$  
D. $\frac{3a}{2}$  
E. $\frac{5a}{3}$

15. The value of $\lim_{h \to 0} \frac{\sqrt{2 + h} - \sqrt{2}}{h}$ is

A. 0  
B. $\frac{1}{2\sqrt{2}}$  
C. $\frac{1}{2}$  
D. $\frac{1}{\sqrt{2}}$  
E. $\infty$
Use of Mathematics Data Bank

Section I: Working tapes

These tapes may be read by the OS of an IBM System/360 series computer. Examples of the required JCL (Job Control Language) cards under ACP and convenient read routines are given below. The data are stored in vectors, each describing a school, a teacher, or a student. A blank vector is used to separate the data of one country from the next. Each tape is ... with a blank vector. Different populations are stored on different...-

Within each country, the data are ordered:

School vector: all teachers for that school; all students for that school;

school vector: ............ etc.

The contents of each vector are described in the Data Bank. The tape format is given in detail in section (c). Vectors are blocked, 50 to a record, and the tape is written (9-track) at 1000 bps.

- Stockholm
(a) Subroutine WREAD

The purpose of this routine is to read all the vectors on a tape in their correct formats. WREAD expects to find the data bank tape on file II.

To use this read routine in an exec program:

CALL WREAD (VEC, ITYPE, INEXT)

Where VEC is a vector (always dimensioned at 124) which returns a record of information for a school, a teacher or a student. ITYPE can be tested to determine the type of vector in VEC, where: if ITYPE:

- 4 national vector (blank)
- 3 school vector
- 2 teacher vector
- 1 student vector
- 0 end of file (tape)

INEXT, set internally by WREAD, indicates the type of the next vector on the file, INEXT uses the same coding convention as ITYPE. This value is used by the routine to select the correct reading format. INEXT must be initialized to 4 before commencing to read a file.

Example: to read and process all vectors one at a time

DIMENSION VEC (124)

INEXT = 4

1 CALL WREAD (VEC, ITYPE, INEXT)

IF (ITYPE .EQ. 0) GO TO 6

GO TO (2, 3, 4, 5), ITYPE

2 ..............

GO TO 1

3 ..............

GO TO 1

4 ..............

GO TO 1

5 ..............

GO TO 1

4. CONTINUE
(b) Subroutine SELVIN

This NONTRAN program (which itself calls WREAD) can be used to select particular types of vector (e.g. schools) from the file. An initial call to SELVIN is required and thereafter calls to SELVEC will produce vectors of the required type.

CALL SELVIN

CALL SELVEC (VEC, ITYPE, &1, &m, &n)

where VEC (dimensioned at 124) returns the required vector. ITYPE defines the vector to be selected using the same code as in WREAD.

1 = is the return statement address for a normal return

m = is the return statement address when the end of a country's data has been reached.

n = is the return address when an end of file condition has been diagnosed by WREAD. The contents of VEC should then be ignored.

Example: to read and process all the school vectors in the first country on the tape

DIMENSION VEC (124)

CALL SELVIN

1 CALL SELVEC (VEC, 3, &2, &3, &3)

2 CONTINUE

3 CONTINUE
### WORKING TAPE FORMAT

<table>
<thead>
<tr>
<th>NATIONAL VECTOR</th>
<th>STUDENT VECTOR</th>
<th>SCHOOL VECTOR</th>
<th>TEACHER VECTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>II</td>
<td>322X</td>
<td>Blank</td>
</tr>
<tr>
<td>1</td>
<td>II</td>
<td>Next rec. type</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>II</td>
<td>As in manual</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>II</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>II</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>II</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>II</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>II</td>
<td></td>
<td>7-9</td>
</tr>
<tr>
<td>8-19</td>
<td>12II</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>T4</td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>21</td>
<td>T3</td>
<td></td>
<td>12-13</td>
</tr>
<tr>
<td>22-25</td>
<td>411</td>
<td></td>
<td>14-17</td>
</tr>
<tr>
<td>26-27</td>
<td>212</td>
<td></td>
<td>16-19</td>
</tr>
<tr>
<td>28-29</td>
<td>215.2</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>30</td>
<td>F4.3</td>
<td></td>
<td>21</td>
</tr>
<tr>
<td>31</td>
<td>F3.2</td>
<td></td>
<td>22</td>
</tr>
<tr>
<td>32-46</td>
<td>1511</td>
<td></td>
<td>23</td>
</tr>
<tr>
<td>47-49</td>
<td>374.2</td>
<td></td>
<td>51-57</td>
</tr>
<tr>
<td>50</td>
<td>T3</td>
<td></td>
<td>58</td>
</tr>
<tr>
<td>51</td>
<td>II</td>
<td>Next record type</td>
<td>59</td>
</tr>
<tr>
<td>23XX</td>
<td>Skip</td>
<td></td>
<td>60-63</td>
</tr>
<tr>
<td>52</td>
<td>II</td>
<td></td>
<td>64-89</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4X</td>
</tr>
<tr>
<td>TEACHER VECTOR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>II</td>
<td>Always 2</td>
<td>0-103</td>
</tr>
<tr>
<td>1</td>
<td>II</td>
<td>As in manual</td>
<td>104-111</td>
</tr>
<tr>
<td>2</td>
<td>II</td>
<td></td>
<td>112-113</td>
</tr>
<tr>
<td>3</td>
<td>II</td>
<td></td>
<td>114-124</td>
</tr>
<tr>
<td>4</td>
<td>II</td>
<td></td>
<td>125</td>
</tr>
<tr>
<td>5</td>
<td>T2</td>
<td></td>
<td>Skip</td>
</tr>
<tr>
<td>6-23</td>
<td>18II</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>F3.1</td>
<td></td>
<td></td>
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<tr>
<td>25-42</td>
<td>18F4.2</td>
<td></td>
<td>5X</td>
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<td>43-49</td>
<td>7F4.2</td>
<td>As in manual</td>
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</tr>
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<td>50-64</td>
<td>1511</td>
<td></td>
<td>172X</td>
</tr>
<tr>
<td>65</td>
<td>II</td>
<td>Next rec. type</td>
<td></td>
</tr>
</tbody>
</table>

Each tape contains one file followed by an end of file mark. The logical records each describing one of the Student, Teacher, School or National vectors, can be read in the order described above. Each record contains 324 characters (bytes) and there are 50 records to a block, thus 16,200 characters per block. Numbers in the floating point format do not contain any decimal point, therefore the number 32.4 would appear on the tape as 324 and would be read as F3.1. Refer to the Data Bank Manual for descriptions of the variables. A new variable has been added, numbered 0, which is first in each record and indicates the type (student, teacher, etc.) of the record. The last variable in each record; also not in the manual, describes the type of the following record. This is for ease in the proper format selection before the record is read. The last variable of the very last record on each tape will be zero.

The file vector on each tape is always a national vector.